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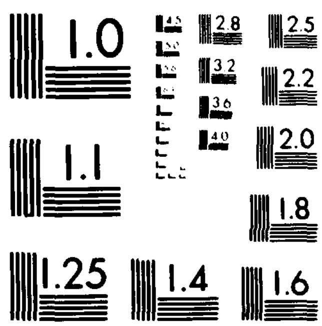

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TECHNICAL REPORT T-79-63

INVESTIGATION OF DISTURBANCE ACCOMMODATING CONTROLLER APPLICATION TO A MISSILE AUTOPILOT

Wayne L. McCowan
Guidance and Control Directorate
Technology Laboratory

U.S. ARMY MISSILE RESEARCH AND DEVELOPMENT COMMAND

31 May 1979

Redstone Arsenal, Alabama 35809

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The Disturbance Accommodation Controller (DAC) uses a combination of waveform-mode disturbance modeling and state-variable control techniques. This approach permits three primary modes of disturbance accommodation: (1) cancellation (absorption) of disturbance effects, (2) minimization of disturbance effects, or (3) constructive utilization of the disturbances as an aid in accomplishing the primary control task. It is possible to minimize the effects of the disturbances modeled in this report by use of DAC techniques. It is also possible to combine two separate DAC's designed for disturbances at different

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20.

places in the plant, into a functioning unit which would still perform its overall purpose. However, the true test of how a DAC would function in a system application would be to implement one in a 6-DOF simulation and fly it with a severe program of varying disturbance vectors.

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1. INTRODUCTION

This report presents the results of an investigation into the feasibility of using Disturbance Accommodating Controller (DAC) design techniques, as developed by Dr. C. D. Johnson of the University of Alabama in Huntsville, to cancel out disturbance inputs to a missile autopilot channel.

The DAC method of design uses a combination of waveform-mode disturbance modeling and state-variable control techniques. As a tool for controller design, the DAC approach permits three primary modes of disturbance accommodation: (1) cancellation (absorption) of disturbance effects, (2) minimization of disturbance effects, or (3) constructive utilization of the disturbances as an aid in accomplishing the primary control task.

The purpose of this report is to determine if these techniques, specifically the cancellation and minimization modes, can be successfully applied to a missile system in such a manner as to cancel out the effects of disturbance inputs which would otherwise degrade system accuracy.

It is not the intent of this report to thoroughly cover all the background theory involved in the development of DAC design procedures. This theory can best be obtained by reading the original papers; see, for instance, *References 1-4*. For applications of DAC to several simple systems see *Reference 5*.

2. SOME BACKGROUND

The plant considered in this report is one which can be described by state equations of the form

$$\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} + \underline{\underline{F}}\,\underline{w} \\ \dot{\underline{y}} &= \underline{\underline{C}}\,\underline{x} + \underline{\underline{E}}\,\underline{u} + \underline{\underline{G}}\,\underline{w} \end{aligned} \tag{1}$$

where

$\underline{x}$ is the plant state vector,
$\underline{u}$ is the plant control input vector,
$\underline{w}$ is the vector of external disturbance acting on the plant,
$\underline{y}$ is the system output vector, and
$\underline{\underline{A}}, \underline{\underline{B}}, \underline{\underline{F}}, \underline{\underline{C}}, \underline{\underline{E}}, \underline{\underline{G}}$ are appropriate size, known matrices which are not necessarily constant.

Now, the external disturbances, w(t), for which DAC theory is intended are characterized by the presence of "waveform structure," i.e., the functions w(t) can be described by known differential equations which the w(t) satisfy "almost everywhere." For the cases considered in this report, the disturbances will be assumed to be described by the following general set of linear disturbance state equations:

$$\underline{w} = \underline{H}\underline{z} + \underline{L}\underline{x}$$

$$\dot{\underline{z}} + \underline{D}\underline{z} + \underline{M}\underline{x} + \underline{\sigma} \tag{2}$$

where

$\underline{z}$ is the disturbance "state" vector,

$\underline{\sigma}$ is a sequence of randomly arriving vector impulses, and

$\underline{D}$, $\underline{H}$, $\underline{L}$, $\underline{M}$ are known, time-invariant matrices.

In most practical applications, neither the complete set of plant state variables nor the various components $w_i(t)$ of the disturbance are available for direct on-line measurement. Therefore, the DAC is restricted to operate only on information in the available on-line measurements of the system outputs and commands and any disturbance components which may actually be available for direct measurement. In the case at hand, it is assumed that none of the disturbance components are measurable on-line and that the information available from the plant consists of the input command, command to the actuators and the measured pitch plane acceleration and rate components of the missile motion.

Since the idealized DAC control law is a function of the real-time system state, $\underline{x}$, and disturbance state, $\underline{z}$, the required on-line data for practical DAC implementation must be generated via use of state reconstructors (observers) operating on real-time system outputs $\underline{y}$ and control inputs $\underline{u}$. Since the external disturbances w(t) are assumed to have waveform structure and to be modeled by known linear state models, a state reconstructor can be designed to generate estimates $\hat{\underline{z}}$ of the instantaneous disturbance state $\underline{z}$. In addition, that same state reconstructor can be designed to produce estimates $\hat{\underline{x}}$ of the instantaneous system state $\underline{x}$.

Procedures have been developed to generate both "full-dimensional" observers of dimension ($n+\rho$), where n is the order of $\underline{x}$ and ρ the order of $\underline{z}$, and "reduced-dimensional" observers of dimension ($n+\rho-m$), where n, ρ are as above and m is the rank of $\underline{C}$. The work performed in this study is concerned with "full-dimensional" observers.

For the form of the state equations given by Equation (1), the full-dimensional observer is expressed as

$$\begin{pmatrix} \dot{\hat{\underline{x}}} \\ \dot{\hat{\underline{z}}} \end{pmatrix} = \left[\begin{array}{c|c} \underline{A} + \underline{F}\underline{L} + \underline{K}_1(\underline{C} + \underline{G}\underline{L}) & [\underline{F} + \underline{K}_1\underline{G}]\underline{H} \\ \hline \underline{M} + \underline{K}_2(\underline{C} + \underline{G}\underline{L}) & \underline{D} + \underline{K}_2\underline{G}\underline{H} \end{array}\right] \begin{pmatrix} \hat{\underline{x}} \\ \hat{\underline{z}} \end{pmatrix} - \left[\begin{array}{c} \underline{K}_1 \\ \hline \underline{K}_2 \end{array}\right] \underline{y}(t)$$

$$+ \left[\begin{array}{c} \underline{B} + \underline{K}_1\underline{E} \\ \hline \underline{K}_2\underline{E} \end{array}\right] \underline{u}(t) \tag{3}$$

where $\underline{K}_1$, $\underline{K}_2$, are gain matrices to be designed,
$\underline{A}$, $\underline{F}$, $\underline{L}$, $\underline{C}$, $\underline{G}$, $\underline{H}$, $\underline{D}$, $\underline{M}$ are as previously described.

Such a composite-type state reconstructor can be utilized to implement DAC control laws in the form

$$u = f(\hat{x},\hat{z},t).$$

Of course, for acceptable performance the real-time estimation errors

$$\underline{\varepsilon}_x = \underline{x} - \hat{\underline{x}}$$
$$\underline{\varepsilon}_z = \underline{z} - \hat{\underline{z}}$$

must settle to zero rapidly in comparison to system settling times where $\underline{\varepsilon}_x$ and $\underline{\varepsilon}_z$ are given by

$$\begin{pmatrix} \dot{\underline{\varepsilon}}_x \\ \dot{\underline{\varepsilon}}_z \end{pmatrix} = \left[\begin{array}{c|c} \underline{A} + \underline{F}\underline{L} + \underline{K}_1(\underline{C} + \underline{G}\underline{L}) & [\underline{F} + \underline{K}_1\underline{G}]\,\underline{H} \\ \hline \underline{M} + \underline{K}_2(\underline{C} + \underline{G}\underline{L}) & \underline{D} + \underline{K}_2\underline{G}\underline{H} \end{array}\right] \begin{pmatrix} \underline{\varepsilon}_x \\ \underline{\varepsilon}_z \end{pmatrix} + \begin{pmatrix} \underline{0} \\ \underline{\sigma}(t) \end{pmatrix} \tag{4}$$

3. PLANT

The plant utilized for the studies detailed in this report is the pitch plane acceleration autopilot channel shown in block diagram form in *Figure 1*. An ideal accelerometer is assumed in the acceleration feedback loop and an ideal rate gyro is assumed in the rate feedback loop. Also, no actuator dynamics are considered.

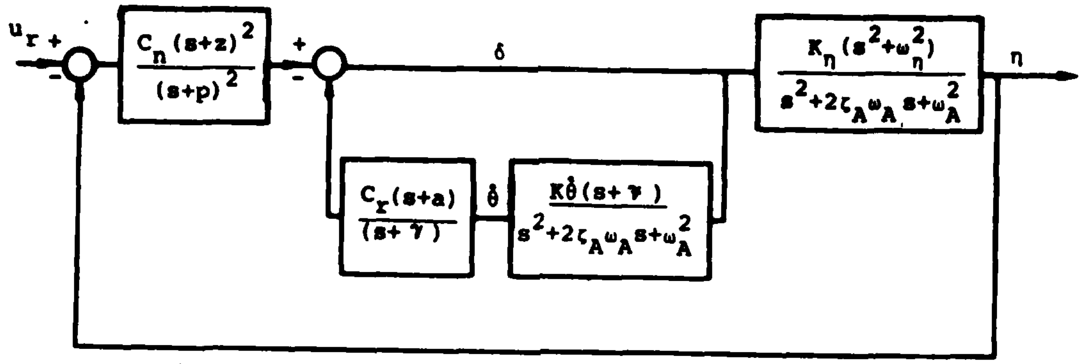


Figure 1. Pitch acceleration autopilot channel.

The transfer functions indicated in *Figure 1* are:

a. Pitch rate per fin deflection in pitch:

$$\frac{\dot\theta(s)}{\delta(s)} = \frac{K_{\dot\theta} \ (s + \gamma)}{s^2 + 2\zeta_A \omega_A s + \omega_A^2}$$

b. Lateral acceleration per fin deflection in pitch:

$$\frac{\eta(s)}{\delta(s)} = \frac{K_\eta (s^2 + \omega_\eta^2)}{s^2 + 2\zeta_A \omega_A s + \omega_A^2} \cdot$$

with the terms in the transfer functions being determined from the aerodynamic characteristics of the missile at given points along a trajectory.

c. Autopilot compensation terms:

$$\frac{C_R(s+a)}{s+\gamma} \quad \text{and} \quad \frac{c_\eta (s+z)^2}{(s+p)^2}$$ are compensation terms

which were designed into the autopilot to improve performance. Most of the terms are varied over a trajectory according to dynamic pressure.

Since the transfer function parameters and most of the autopilot compensation terms do vary along the missile trajectory, several representative points along a nominal trajectory where chosen as design points for use in this report. These points were chosen to cover as nearly as possible the entire range of values of the parameters involved. *Table I* lists the time points and parameter values.

For the initial investigation, it was decided to look at several different configurations involving the plant, or some part of it, and a disturbance source. First, the entire loop was used with a disturbance assumed to be acting at the input. Next, the rate loop alone was considered with an assumed disturbance summed in with the $\dot{\theta}$ due to fin deflection. Third, the entire loop was again used, this time with a disturbance summed into the output. As a final case, the entire loop was used with disturbances at the input and the output. Each of these cases will be detailed later in this report.

4. DISTURBANCE MODEL

The disturbances modeled here in all cases were taken to be composed of constants plus ramps, i.e.,

$$w(t) = C_0 + C_1 t \tag{5}$$

where C_0 and C_1 are, in general, unknown a priori and can change value in a completely unknown random-like manner. This disturbance model was chosen because it is easy to work with but still illustrates the point.

To put (5) into the form (2), proceed as follows. First, take the Laplace Transform of w(t),

$$w(s) = \frac{C_0}{s} + \frac{C_1}{s^2} = \frac{C_0 s + C_1}{s^2}$$

The characteristic polynomial associated with this is

$$\lambda^2 = 0 \; . \tag{6}$$

TABLE 1. AIRFRAME/COMPENSATION PARAMETERS

PARAMETER \ FLIGHT TIME (SEC)	9.85 (JUST AFTER BURNOUT)	18.0	50.5	66.7 (APOGEE)	103.3	111.4	135.8
ζ_A	0.0256	0.02	0.01	0.009	0.014	0.017	0.038
ω_A	14.54	8.7	2.216	1.77	3.87	5.21	9.48
$K_{\dot{\theta}}$	-107.	-50.	-6.56	-5.17	-14.68	-25.4	-118.6
γ	0.536	0.255	0.034	0.026	0.08	0.138	0.56
K_η	-317.8	-148.5	-19.5	-15.35	-43.6	-75.3	-352.2
ω_η^2	-540	-209.9	-18.16	-12.9	-48.	-85.6	-330.4
C_n	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$
z	10.	10.	10.	10.	10.	10.	10.
p	1.	1.	1.	1.	1.	1.	1.
C_R	-0.1396	-0.1396	-0.4363	-0.4363	-0.4363	-0.4363	-0.4363

Therefore, choose

$$\underline{w} = \underline{\underline{H}}\underline{z}$$
$$\dot{\underline{z}} = \underline{\underline{D}}\underline{z} + \underline{\sigma}$$

(Note: no state dependence terms are included in this case) such that

$$\dot{\underline{z}} = \underline{\underline{D}}\underline{z}$$

has a characteristic polynomial $\lambda^2 = 0$ and $\underline{\underline{H}}\underline{z}$ has the general form $w = C_0 + C_1 t$.

So, let

$$\underline{w} = \underline{\underline{H}}\underline{z} = (1 \quad 0) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

and

$$\dot{\underline{z}} = \underline{\underline{D}}\underline{z} + \underline{\sigma} = \begin{bmatrix} -\beta_2 & 1 \\ -\beta_1 & 0 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \underline{\sigma} \ .$$

Then,

$$\det \left| \underline{\underline{D}} - \lambda \underline{\underline{I}} \right| = \begin{vmatrix} -\beta_2 - \lambda & 1 \\ -\beta_1 & -\lambda \end{vmatrix} = \lambda^2 + \beta_2 \lambda + \beta_1 = 0 \ . \qquad (7)$$

Comparing (7) with (6), one must have $\beta_2 = \beta_1 = 0$.

Thus,

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \underline{\sigma}$$

$$\underline{w} = (1 \quad 0) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \qquad (8)$$

From this one has, therefore,

$$\underline{H} = (1 \;\; 0)$$

$$\underline{D} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} .$$

These two matrices are used throughout this report in the disturbance state model.

In order to give a feel for just what "disturbance" as an entity is insofar as the applications here are concerned, it could be any or all of: wind or wind gust, thrust misalignment, tipoff rates, biases, instrument drifts, target motion and more. An influencing agent which has waveform structure and which imposes an undesirable effect on the system may be considered a disturbance.

5. ACCELERATION LOOP WITH DISTURBANCE AT INPUT

A. DAC MODEL DEVELOPMENT

A block diagram representation of the autopilot/disturbance combination used in the development for this section is shown in *Figure 2*.

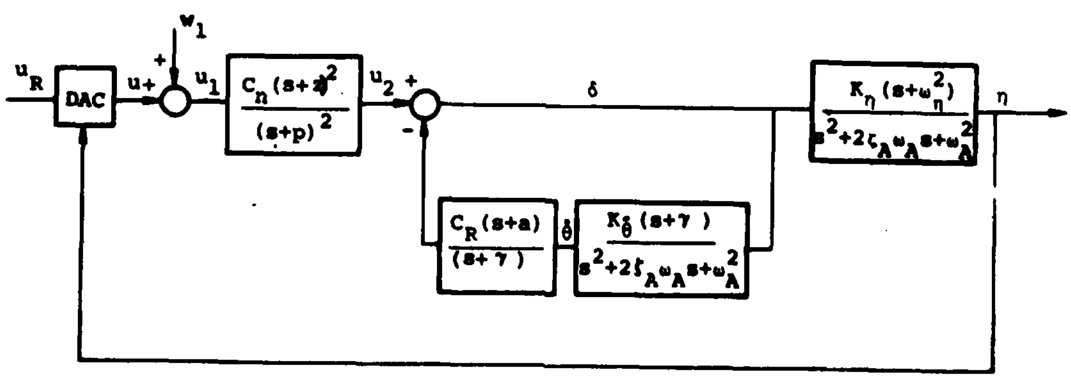


Figure 2. Pitch acceleration channel with disturbance at input.

The closed loop transfer function for the rate loop is

$$\frac{\delta(s)}{u_2(s)} = \frac{s^2 + 2\zeta_A\omega_A s + \omega_A^2}{s^2 + (2\zeta_A\omega_A + K_{\dot\theta}C_R)s + (\omega_A^2 + K_{\dot\theta}C_R)} \tag{9}$$

Let

$$a_0 = 2\zeta_A\omega_A$$

$$a_1 = 2\zeta_A\omega_A + K_{\dot\theta}C_R$$

$$a_2 = \omega_A^2 + K_{\dot\theta}C_R \quad .$$

Then

$$\frac{\delta(s)}{u_2(s)} = \frac{s^2 + a_0 s + \omega_A^2}{s^2 + a_1 s + a_2} \quad . \tag{10}$$

With this, the product of the transfer function blocks between u_1 and η is

$$\frac{K_\eta C_n (s + z)^2 (s^2 + \omega_\eta^2)}{(s + p)^2 (s^2 + a_1 s + a_2)} = \frac{K_\eta C_n \left[s^4 + 2zs^3 + (z^2 + \omega_\eta^2)s^2 + 2z\omega_\eta^2 s + \omega_\eta^2 z^2\right]}{s^4 + (2p + a_1)s^3 + (p^2 + 2pa_1 + a_2)s^2 + (a_1p^2 + 2pa_2)s + a_2p^2} \quad . \tag{11}$$

Let

$$b_0 = 2z$$

$$b_1 = z^2 + \omega_\eta^2$$

$$b_2 = 2z\omega_\eta{}^2$$

$$b_3 = \omega_\eta{}^2 z^2$$

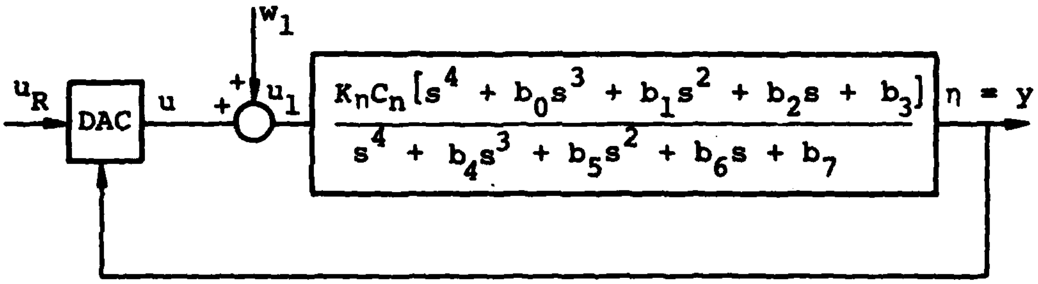
$$b_4 = 2p + a_1$$

$$b_5 = p^2 + 2pa_1 + a_2$$

$$b_6 = a_1p^2 + 2pa_2$$

$$b_7 = a_2p^2$$

The block diagram has thus been reduced to



In order to represent the plant in the form (1), proceed as follows.

$$\frac{y(s)}{u_1(s)} = \frac{K_\eta C_n\ [s^4 + b_0s^3 + b_1s^2 + b_2s + b_3]}{s^4 + b_4s^3 + b_5s^2 + b_6s + b_7}$$

Cross-multiplying gives

$$[s^4 + b_4s^3 + b_5s^2 + b_6s + b_7]\ y(s) = K_\eta C_n\ [s^4 + b_0s^3 + b_1s^2 + b_2s + b_3]\ u_1(s)$$

Solving for y(s),

$$y(s) = K_\eta C_n u_1 + \frac{1}{s}\Big\langle K_\eta C_n b_o u_1(s)\ b_4 y(s) + \frac{1}{s}\Big\{ K_\eta C_n b_1 u_1(s)$$

$$- b_5 y(s) + \frac{1}{s}\,[K_\eta C_n b_2 u_1(s) - b_6 y(s)$$

$$+ \frac{1}{s}\,(K_\eta C_n b_3 u_1(s) - b_7 y(s))]\Big\}\Big\rangle \tag{12}$$

where $\frac{1}{s}$ denotes an integration.

This can be represented diagrammatically as

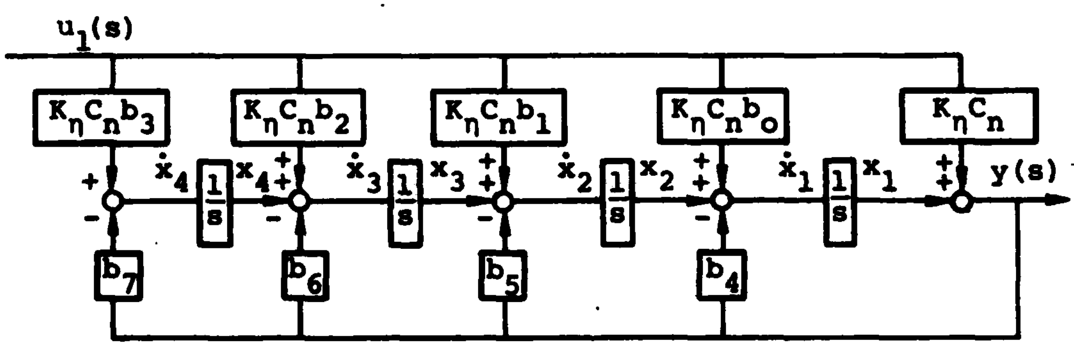


Figure 3. Plant state representation.

and from this, the equations for the states can be written directly as

$$\dot{x}_1 = x_2 + K_\eta C_n b_0 u_1 - b_4 y$$

$$\dot{x}_2 = x_3 + K_\eta C_n b_1 u_1 - b_5 y$$

$$\dot{x}_3 = x_4 + K_\eta C_n b_2 u_1 - b_6 y$$

$$\dot{x}_4 = K_\eta C_n b_3 u_1 - b_7 y$$

$$y = x_1 + K_\eta C_n u_1$$

However, for purposes of DAC design these equations need to be expressed as functions of $\underline{x}$, $\underline{u}$ and $\underline{w}$. So, since $u_1 = u + w_1$,

$$y = x_1 + K_\eta C_n \; (u+w_1)$$

$$\dot{x}_1 = -b_4 x_1 + x_2 + K_\eta C_n (u+w_1) \; (b_0-b_4)$$

$$\dot{x}_2 = -b_5 x_1 + x_3 + K_\eta C_n (u+w_1) \; (b_1-b_5)$$

$$\dot{x}_3 = -b_6 x_1 + x_4 + K_\eta C_n (u+w_1) \; (b_2-b_6)$$

$$\dot{x}_4 = -b_7 x_1 + K_\eta C_n (u+w_1) \; (b_3-b_7) \quad . \tag{13}$$

or, in matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -b_4 & 1 & 0 & 0 \\ -b_5 & 0 & 1 & 0 \\ -b_6 & 0 & 0 & 1 \\ -b_7 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + K_\eta C_n \begin{bmatrix} b_0-b_4 \\ b_1-b_5 \\ b_2-b_6 \\ b_3-b_7 \end{bmatrix} \underline{u}$$

$$+ K_\eta C_n \begin{bmatrix} b_0-b_4 \\ b_1-b_5 \\ b_2-b_6 \\ b_3-b_7 \end{bmatrix} \underline{w}_1 \quad . \tag{14}$$

$$\underline{y} = [1\ 0\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [K_\eta C_n]\ \underline{u} + [K_\eta C_n]\ \underline{w}_1 \tag{15}$$

Thus, Equations (14) and (15) define the remainder of the matrices needed for the DAC design.

Before proceeding, it is necessary to first check for the existence of a control, u_c, which can totally counteract all the disturbance effects.

This control will exist if and only if $\underline{F} \equiv \underline{B}\ \underline{\Gamma}$ for some $\underline{\Gamma}$. Here,

$$K_\eta C_n \begin{bmatrix} b_0-b_4 \\ b_1-b_5 \\ b_2-b_6 \\ b_3-b_7 \end{bmatrix} \equiv K_\eta C_n \begin{bmatrix} b_0-b_4 \\ b_1-b_5 \\ b_2-b_6 \\ b_3-b_7 \end{bmatrix} \underline{\Gamma} \qquad \text{for } \underline{\Gamma} = 1.$$

Such a control does, therefore, exist and will be

$$\underline{u}_c = -\underline{\Gamma}\ \underline{w}_1 = -w_1 = -\hat{z}_1.$$

Now, a full-dimensional composite state reconstructor in the form of Equation 3 must be designed to provide $\hat{\underline{z}}_1$. Starting with the error dynamics (Equation 4) in order to obtain $\underline{K}_1$ and $\underline{K}_2$ we have (since $\underline{L} = \underline{M} = 0$)

$$\begin{pmatrix} \dot{\underline{\varepsilon}}_x \\ \dot{\underline{\varepsilon}}_z \end{pmatrix} = \left[\begin{array}{c|c} \underline{A} + \underline{K}_1\ \underline{C} & (\underline{F} + \underline{K}_1\ \underline{G})\ \underline{H} \\ \hline \underline{K}_2\underline{C} & \underline{D} + \underline{K}_2\ \underline{GH} \end{array}\right] \begin{pmatrix} \underline{\varepsilon}_x \\ \underline{\varepsilon}_z \end{pmatrix} + \begin{pmatrix} \underline{0} \\ \hline \underline{\sigma} \end{pmatrix}.$$

Substituting in the the appropriate matrix values:

$$\underline{\dot{\varepsilon}} = \left[\begin{array}{c|c} \begin{bmatrix} -b_4 & 1 & 0 & 0 \\ -b_5 & 0 & 1 & 0 \\ -b_6 & 0 & 0 & 1 \\ -b_7 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} k_{11} & 0 & 0 & 0 \\ k_{21} & 0 & 0 & 0 \\ k_{31} & 0 & 0 & 0 \\ k_{41} & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} f_{11} + K_\eta C_n k_{11} \\ f_{21} + K_\eta C_n k_{21} \\ f_{31} + K_\eta C_n k_{31} \\ f_{41} + K_\eta C_n k_{41} \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \\ \hline \begin{bmatrix} k_{12} & 0 & 0 & 0 \\ k_{22} & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} K_\eta C_n k_{12} & 1 \\ K_\eta C_n k_{22} & 0 \end{bmatrix} \end{array}\right] \underline{\varepsilon}$$

$$+ \begin{bmatrix} \underline{o} \\ \underline{\sigma} \end{bmatrix}$$

where

k_{11}, k_{21}, k_{31}, k_{41}, are the elements of $\underline{K}_1$,

k_{12}, k_{22}, are the elements of $\underline{K}_2$ and

f_{11}, f_{21}, f_{31}, f_{41} are the elements of $\underline{F}$.

Performing the matrix addition and multiplication indicated,

$$\underline{\dot{\varepsilon}} = \begin{bmatrix} (k_{11}-b_4) & 1 & 0 & 0 & (f_{11} + K_\eta C_n k_{11}) & 0 \\ (k_{21}-b_5) & 0 & 1 & 0 & (f_{21} + K_\eta C_n k_{21}) & 0 \\ (k_{31}-b_6) & 0 & 0 & 1 & (f_{31} + K_\eta C_n k_{31}) & 0 \\ (k_{41}-b_7) & 0 & 0 & 0 & (f_{41} + K_\eta C_n k_{41}) & 0 \\ k_{12} & 0 & 0 & 0 & K_\eta C_n k_{12} & 1 \\ k_{22} & 0 & 0 & 0 & K_\eta C_n k_{22} & 0 \end{bmatrix} \underline{\varepsilon} + \begin{bmatrix} \underline{o} \\ \underline{\sigma} \end{bmatrix} \qquad (16)$$

To simplify notation in the following development, let Equation (16) be represented as

$$\underline{\dot{\varepsilon}} = \underline{\tilde{A}}\ \underline{\varepsilon} + \begin{bmatrix}\underline{0}\\ \underline{\sigma}\end{bmatrix} \quad \text{and let } \underline{\tilde{A}} \text{ be represented as}$$

$$\underline{\tilde{A}} = \begin{bmatrix} e_0 & 1 & 0 & 0 & e_6 & 0 \\ e_1 & 0 & 1 & 0 & e_7 & 0 \\ e_2 & 0 & 0 & 1 & e_8 & 0 \\ e_3 & 0 & 0 & 0 & e_9 & 0 \\ e_4 & 0 & 0 & 0 & e_{10} & 1 \\ e_5 & 0 & 0 & 0 & e_{11} & 0 \end{bmatrix} .$$

Now, to solve for the gain matrices $\underline{K}_1$ and $\underline{K}_2$, one must first find the eigenvalues of $\underline{\tilde{A}}$.

$$\det\ |\underline{\tilde{A}}-\lambda\underline{I}| = \underline{0}.$$

$$\det\ |\underline{\tilde{A}}-\lambda\underline{I}| = \begin{vmatrix} e_0-\lambda & 1 & 0 & 0 & e_6 & 0 \\ e_1 & -\lambda & 1 & 0 & e_7 & 0 \\ e_2 & 0 & -\lambda & 1 & e_8 & 0 \\ e_3 & 0 & 0 & -\lambda & e_9 & 0 \\ e_4 & 0 & 0 & 0 & e_{10}-\lambda & 1 \\ e_5 & 0 & 0 & 0 & e_{11} & -\lambda \end{vmatrix} = \underline{0}$$

Expanding this determinant about the first column results in the expression

$$\begin{aligned} \det\ |\underline{\tilde{A}}-\lambda\underline{I}| = {} & \lambda^6 - (e_0 + e_{10})\ \lambda^5 + (e_0e_{10}-e_{11}-e_1-e_4e_6)\ \lambda^4 \\ & + (e_0e_{11} + e_1e_{10}-e_2-e_4e_7-e_5e_6)\ \lambda^3 \\ & + (e_1e_{11} + e_2e_{10}-e_3-e_4e_8-e_5e_7)\ \lambda^2 \\ & + (e_2e_{11} + e_3e_{10}-e_4e_9-e_5e_8)\ \lambda \\ & + (e_3e_{11}-e_5e_9)\ . \end{aligned} \qquad (17)$$

If the desired roots of Equation (17) are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$, then the desired characteristic equation is

$$(\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)(\lambda-\lambda_4)(\lambda-\lambda_5)(\lambda-\lambda_6) = 0 \tag{18}$$

Expanding Equation (18) and equating coefficients of like powers of λ between Equations (17) and (18) we see that

(a) $$e_0 + e_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = A_0$$

(b) $$e_0e_{10}-e_{11}-e_1-e_4e_6 = \sum_{i=1}^{5} \sum_{j=i+1}^{6} \lambda_i\lambda_j = A_1$$

(c) $$e_0e_{11} + e_1e_{10}-e_2-e_4e_7-e_5e_6 =$$

$$-\left[\sum_{i=1}^{4} \sum_{j=i+1}^{5} \sum_{k=j+1}^{6} \lambda_i\lambda_j\lambda_k\right] = -A_2$$

(d) $$e_1e_{11} + e_2e_{10}-e_3-e_4e_8-e_5e_7 =$$

$$\sum_{i=1}^{3} \sum_{j=i+1}^{4} \sum_{k=j+1}^{5} \sum_{l=k+1}^{6} \lambda_i\lambda_j\lambda_k\lambda_l = A_3$$

(e) $$e_2e_{11} + e_3e_{10}-e_4e_9-e_5e_8 =$$

$$-\left[\lambda_1\lambda_2\lambda_3\lambda_4(\lambda_5+\lambda_6) + \lambda_1\lambda_2\lambda_5\lambda_6(\lambda_3 + \lambda_4)\right.$$

$$\left. + \lambda_3\lambda_4\lambda_5\lambda_6(\lambda_1 + \lambda_2)\right] = -A_4$$

(f) $$e_3e_{11}-e_5e_9 = \lambda_1\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6 = A_5 \ .$$

Substituting the relations for e_0 through e_{11} from Equation (16) into (a) through (f) and solving for the elements of $\underline{K}_1$ and $\underline{K}_2$, we obtain

$$
\begin{aligned}
k_{11} &= -K_\eta C_n k_{12} + b_4 + A_0 \\
k_{21} &= -K_\eta C_n\ (b_0 k_{12} + k_{22}) + b_5 - A_1 \\
k_{31} &= -K_\eta C_n\ (b_0 k_{22} + b_1 k_{12}) + b_6 + A_2 \\
k_{41} &= -K_\eta C_n\ (b_2 k_{12} + b_1 k_{22}) + b_7 - A_3 \\
k_{12} &= (-b_2 K_\eta C_n k_{22} + A_4) \ / \ K_\eta C_n b_3 \\
k_{22} &= -A_5 \ / \ K_\eta C_n b_3
\end{aligned}
\tag{19}
$$

It is desirable that $\epsilon(t) \rightarrow 0$ rapidly, thus the characteristic roots λ_1, λ_2, λ_3, λ_4, λ_5, λ_6 can be picked to best accomplish this depending on the problem at hand. Having picked the λ's, the gain values (Equation 19) can then be calculated. The full-dimensional observer can now be implemented, giving

$$
\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \\ \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \end{bmatrix}
=
\begin{bmatrix}
(k_{11}-b_4) & 1 & 0 & 0 & K_\eta C_n(k_{11}+b_0-b_4) & 0 \\
(k_{21}-b_5) & 0 & 1 & 0 & K_\eta C_n(k_{21}+b_1-b_5) & 0 \\
(k_{31}-b_6) & 0 & 0 & 1 & K_\eta C_n(k_{31}+b_2-b_6) & 0 \\
(k_{41}-b_7) & 0 & 0 & 0 & K_\eta C_n(k_{41}+b_3-b_7) & 0 \\
k_{12} & 0 & 0 & 0 & K_\eta C_n k_{12} & 1 \\
k_{22} & 0 & 0 & 0 & K_\eta C_n k_{22} & 0
\end{bmatrix}
\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{z}_1 \\ \hat{z}_2 \end{bmatrix}
$$

$$
-\begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \\ k_{41} \\ k_{12} \\ k_{22} \end{bmatrix} \underline{y}
+
\begin{bmatrix}
K_\eta C_n(k_{11}+b_0-b_4) \\
K_\eta C_n(k_{21}+b_1-b_5) \\
K_\eta C_n(k_{31}+b_2-b_6) \\
K_\eta C_n(k_{41}+b_3-b_7) \\
K_\eta C_n k_{12} \\
K_\eta C_n k_{22}
\end{bmatrix} \underline{u} \, .
\tag{20}
$$

Figure 4 is a diagram of the composite plant-DAC system. On the diagram, h_1 - h_5 are the components of the last matrix on the right-hand side of (Equation 20). The remainder of the symbols have been previously defined.

So now we have a disturbance cancelling control term, $u_c = -\hat{z}_1$, and we have a composite state reconstructor which gives $\hat{z}_1$. The questions now are:

- Can the λ's be picked so that ϵ_x and ϵ_z settle to zero "rapidly"?
- If so, does u_c really cancel out the effects of w_1? If both of these can be answered in the affirmative, then

  —How well does the DAC work if the plant parameters are varied from the design point?

  —How do the DAC characteristics vary over the trajectory?

Simulation results should provide answers to these questions.

B. SIMULATION AND RESULTS

The composite system shown in *Figure 4* was simulated on a digital computer. The simulation was written so that the plant parameters could be arbitrarily varied around the point for which the DAC was designed. A listing of this simulation is given in Appendix A.

As a first cut at seeing how effective the DAC would be, several runs were made for the t = 9.85 sec and t = 18 sec points. *Figures 5* and *6* give the results. As can be seen, the DAC effectively cancels out an input disturbance (w_1 = 1.0) equal to the input command.

To check the sensitivity of the DAC to plant parameter variations, a series of runs were made with parameters varied around the t = 18 sec values. *Table 2* is a summary of the results obtained and *Figures 7* through *40* give the system output, y, and reconstructor state, $\hat{z}_1$ (disturbance estimate), for each case. The table shows how the peak value of y varied and how the peak value and settling time of $\hat{z}_1$ varied due to both individual and collective parameter changes. In all cases, the input command is 1 and the settling time is defined to be the time at which the response stays within ±5% of steady-state.

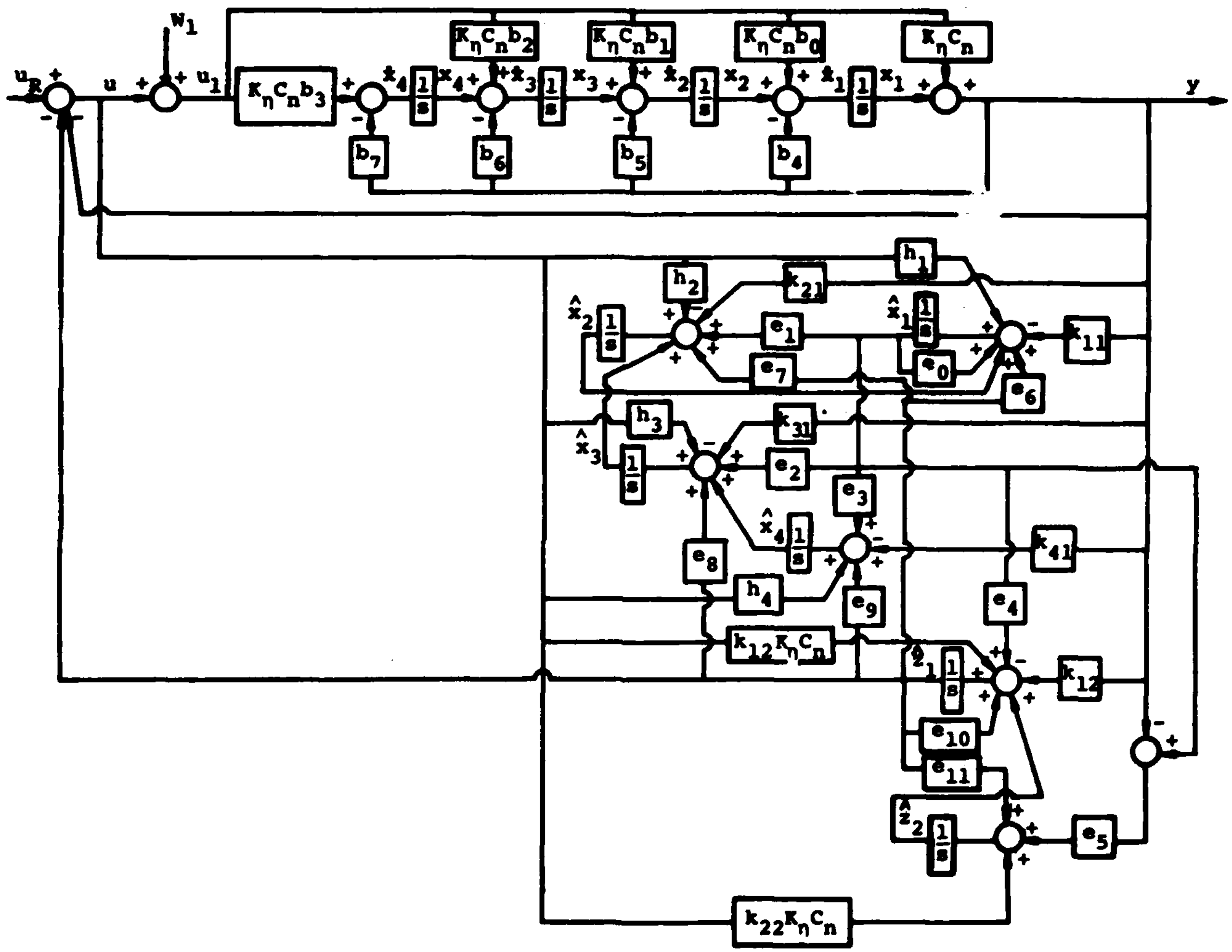


Figure 4. Plant-DAC composite diagram for acceleration loop with disturbance at input.

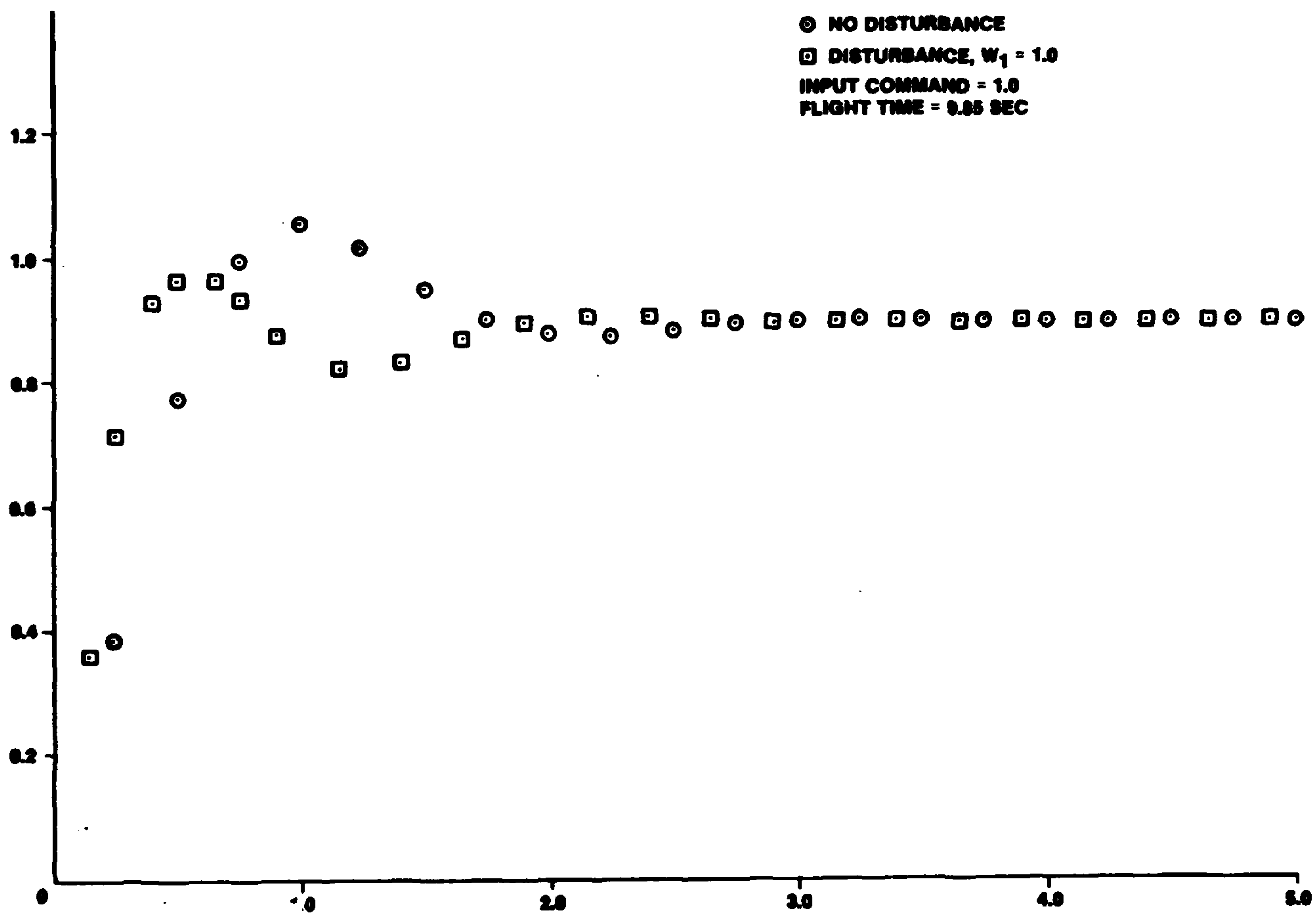


Figure 5. System outputs for t = 9.85 sec case, with and without disturbance.

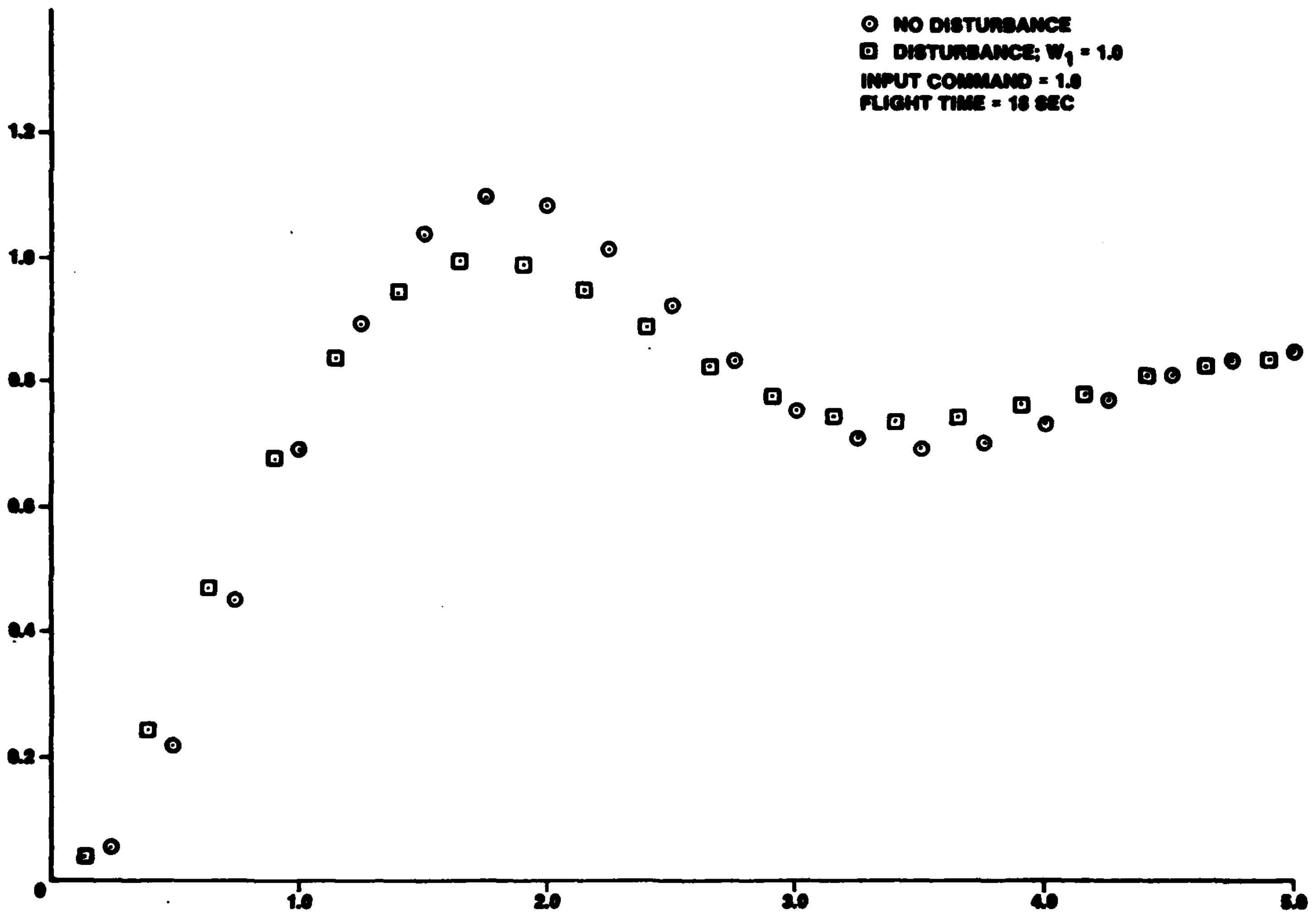


Figure 6. System outputs for t = 18 sec case, with and without disturbance.

TABLE 2. RESPONSE SENSITIVITY TO PLANT PARAMETER VARIATIONS

CONDITION	OUTPUT (y)		DISTURBANCE RECONSTRUCTOR ($\hat{z}_1$)		
	PEAK AMPL	TIME (SEC)	PEAK AMPL	TIME (SEC)	SETTLING TIME
NO DISTURBANCE (NOM)	1.097	1.82	—		—
WITH $W_1 = 1$.:					
· NOMINAL	0.997	1.76	1.35	0.42	1.09
· + 10% $K_{\dot{\theta}}$	0.999	1.725	1.29	0.50	1.32
· - 10% $K_{\dot{\theta}}$	0.994	1.78	1.47	0.35	0.98
· + 10% γ	0.996	1.75	1.36	0.42	1.10
· - 10% γ	0.996	1.75	1.36	0.42	1.10
· + 10% ζ_A	0.99	1.80	1.36	0.45	1.15
· - 10% ζ_A	0.99	1.75	1.36	0.45	1.16
· + 10% ω_A	1.02	1.75	1.2	0.42	5.0
· - 10% ω_A	0.98	1.75	1.54	0.50	4.5
· + 10% $\omega_\eta 2$	0.987	1.78	1.49	0.40	1.18
· - 10% $\omega_\eta 2$	1.01	1.72	1.21	0.48	1.00
· + 10% C_R	1.00	1.75	1.3	0.50	1.4
· - 10% C_R	0.99	1.80	1.47	0.38	1.05
· + 10% K_η	0.98	1.80	1.50	0.40	1.20
· - 10% K_η	1.01	1.70	1.21	0.50	1.05
· + 10% ON ALL	0.99	1.75	1.32	0.45	1.12
· - 10% ON ALL	0.98	1.75	1.45	0.45	1.15
· + 20% ON ALL	0.99	1.8	1.27	0.47	1.1
· - 20% ON ALL	0.98	1.75	1.58	0.45	1.2

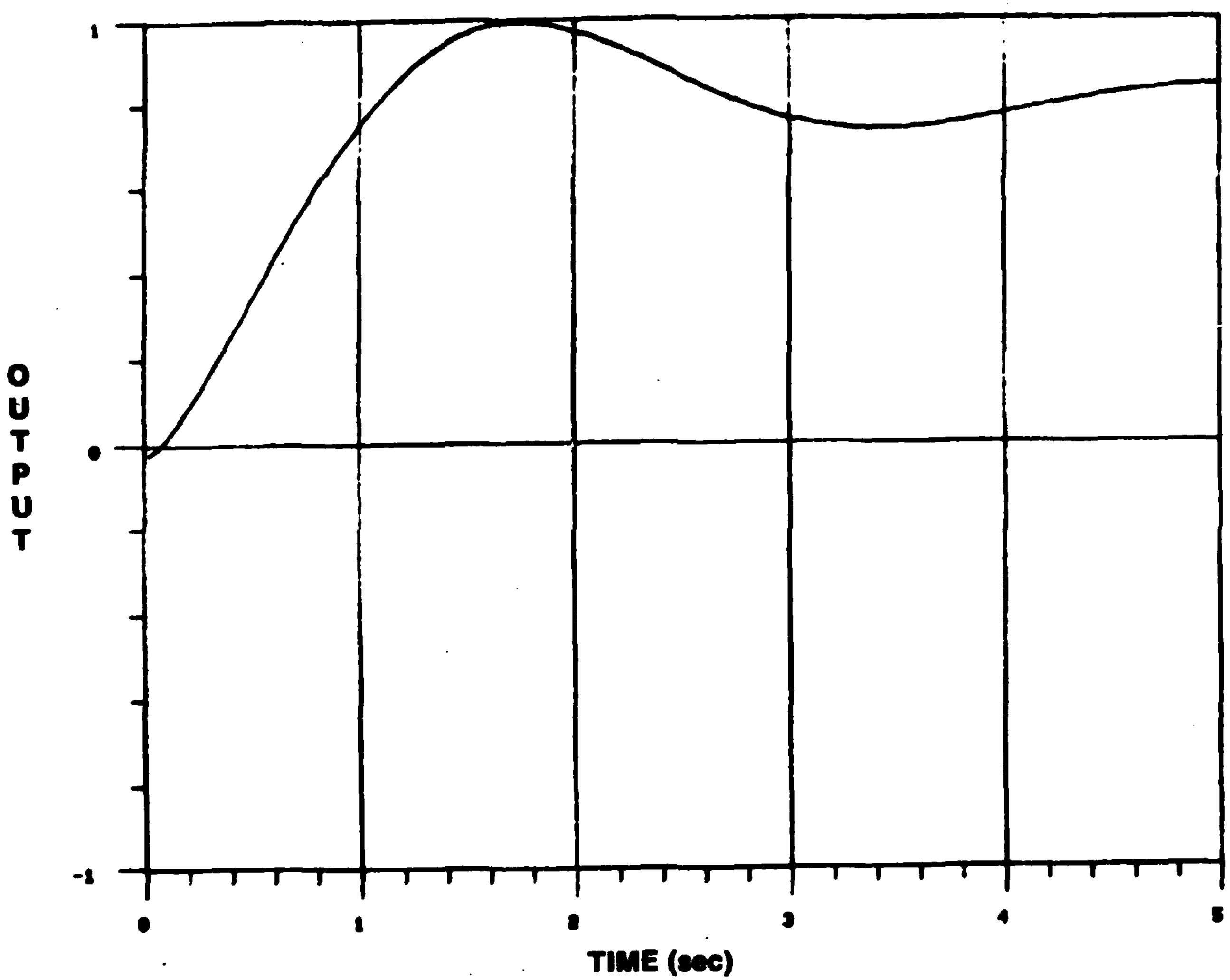


Figure 7. System output response (y), $W_1 = 1$, nominal parameters.

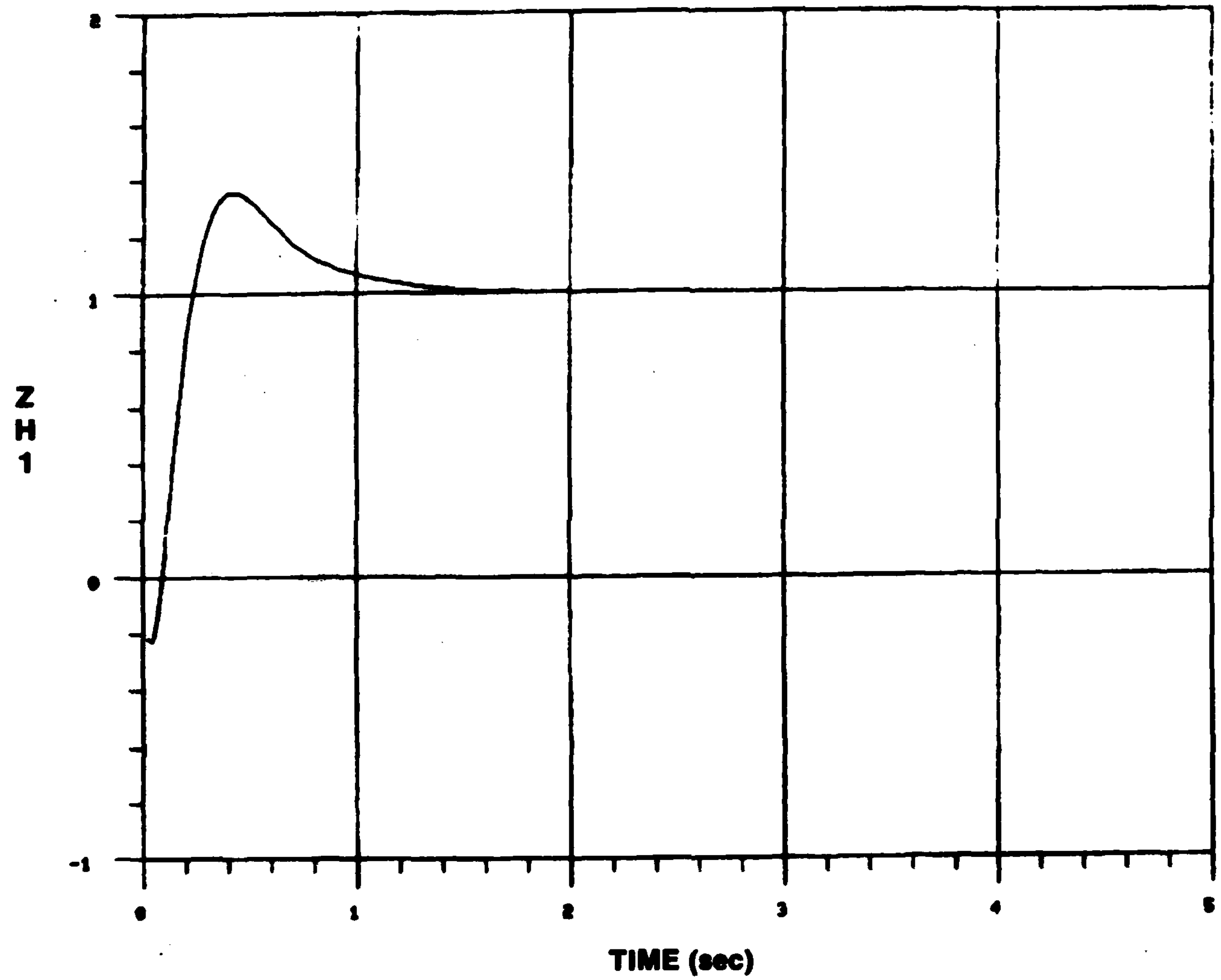


Figure 8. DAC disturbance estimate ($\hat{z}_1$), $W_1 = 1$, nominal parameters.

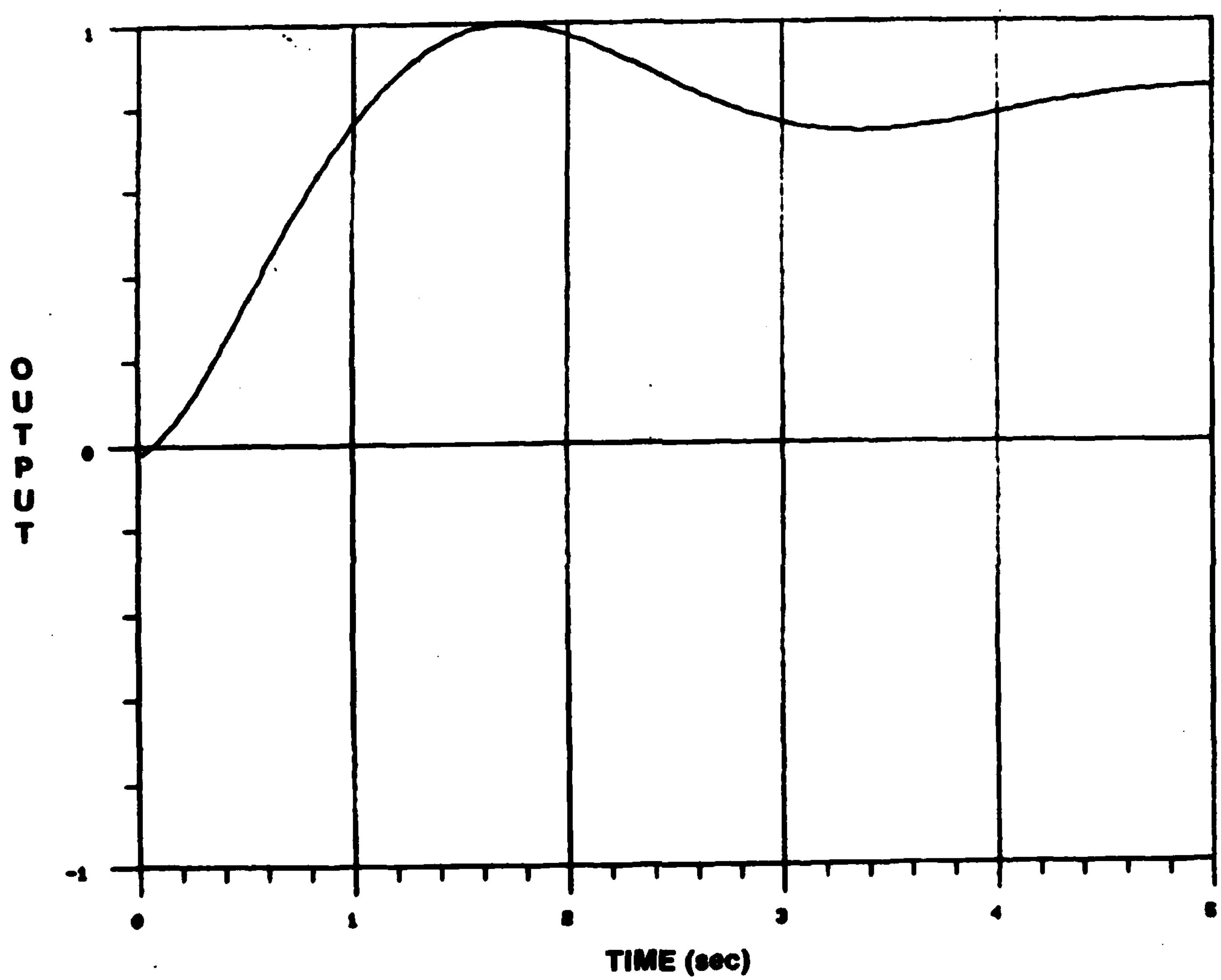


Figure 9. System output response (y), W_1 = 1, +10% variation on $K_{\dot{\phi}}$.

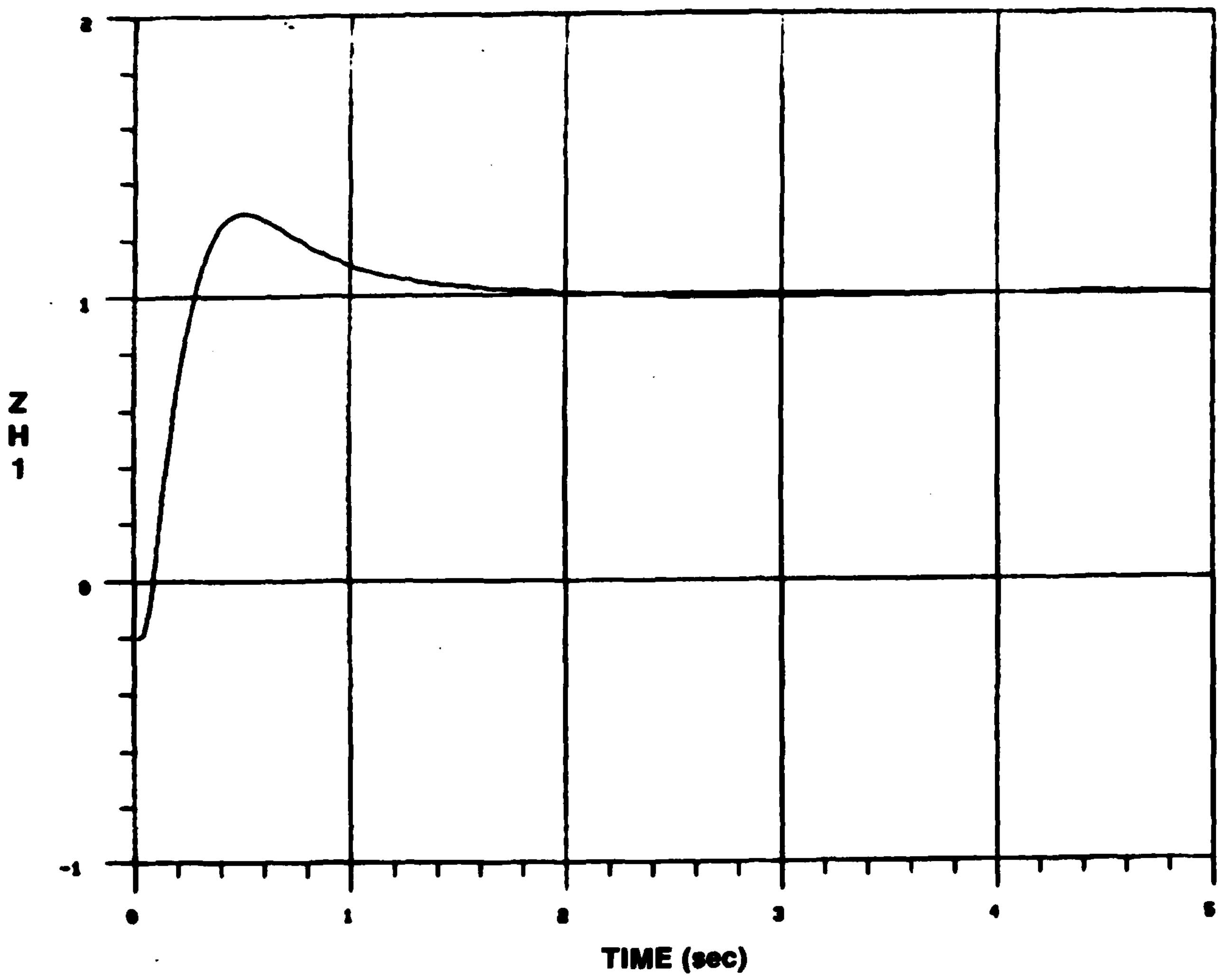


Figure 10. DAC disturbance estimate ($\hat{z}_1$), $W_1 = 1$, +10% variation on $K_{\dot{\theta}}$.

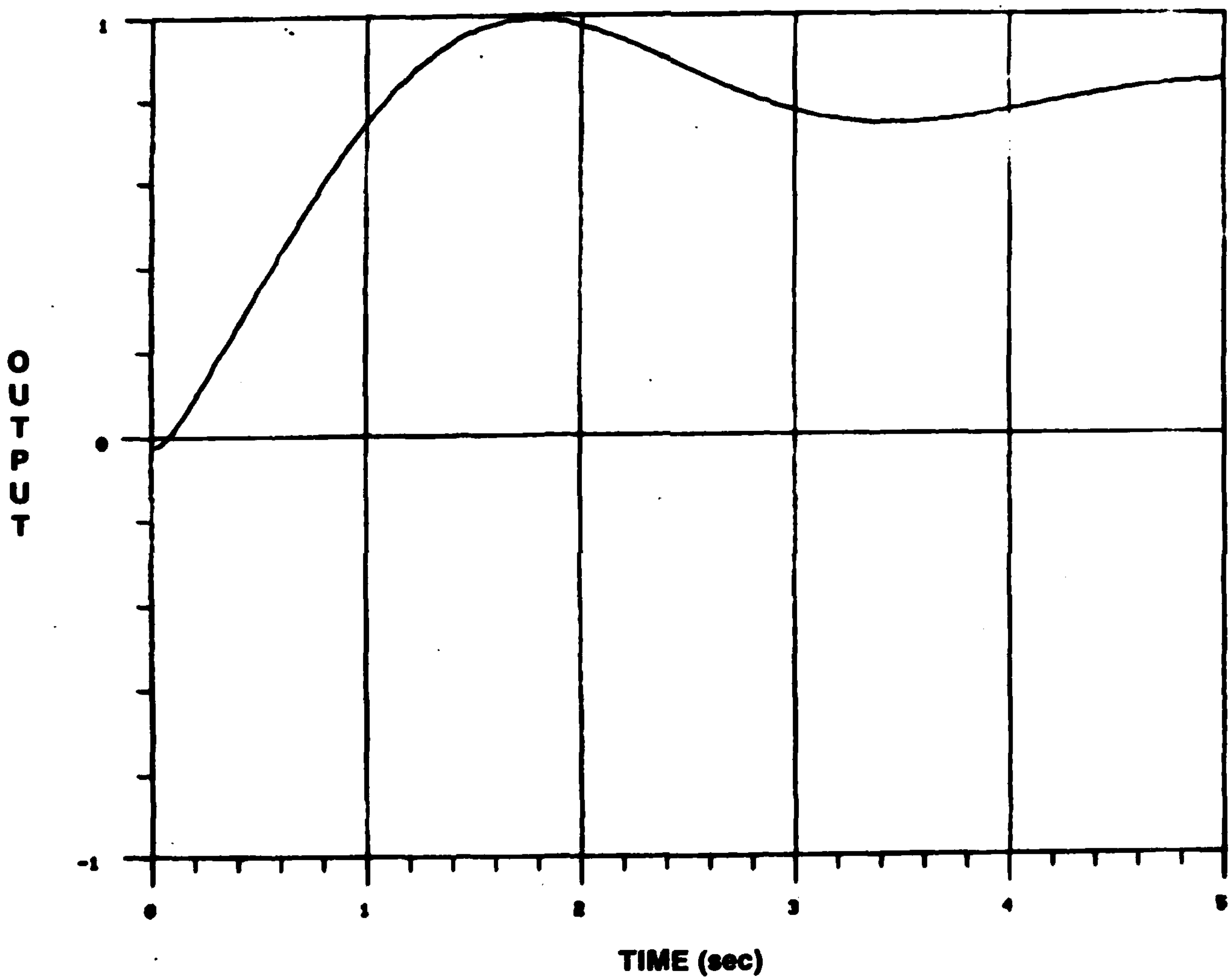


Figure 11. System output response (y), W_1 = 1, -10% variation on $K_{\dot{\theta}}$.

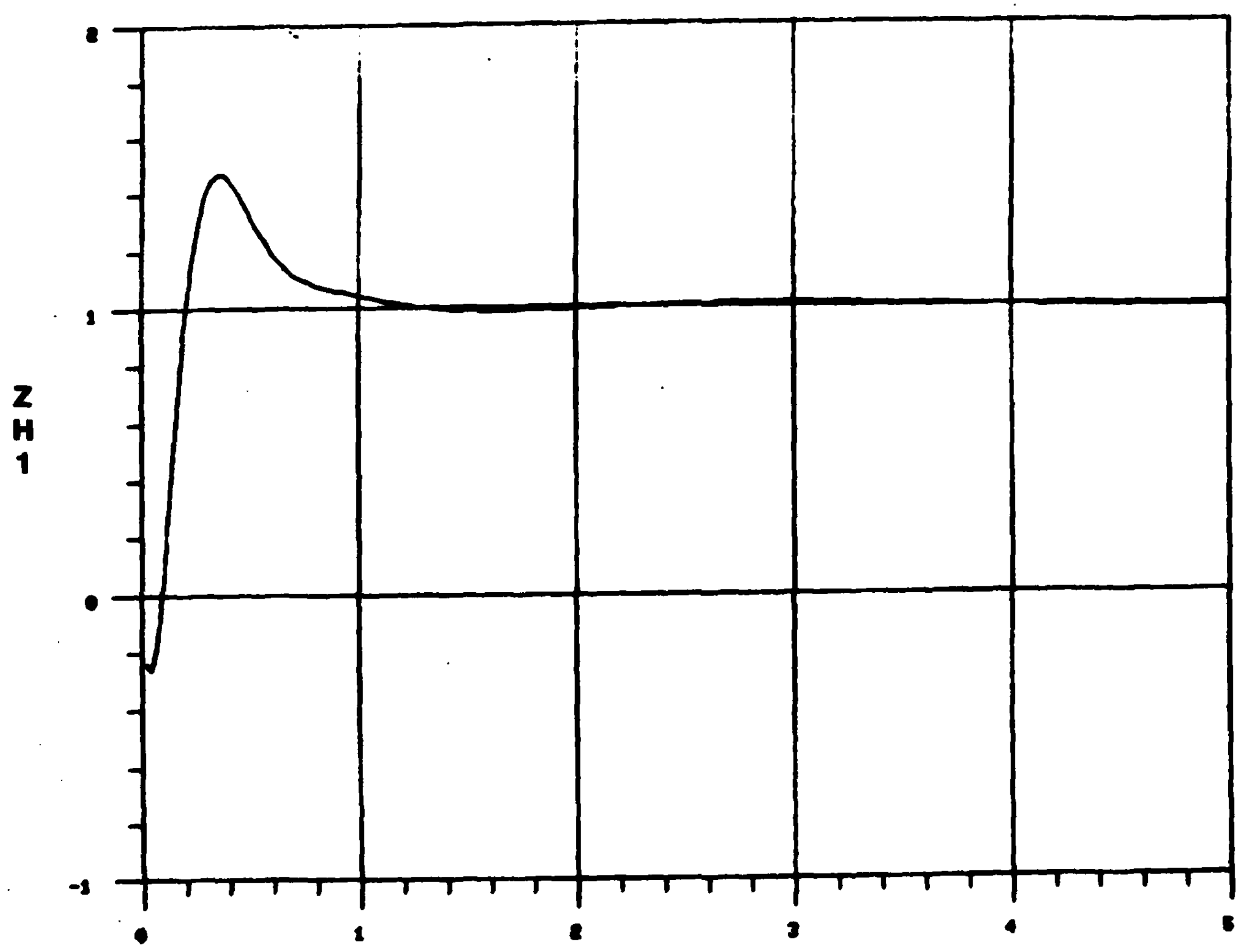


Figure 12. DAC disturbance estimate ($\hat{z}_1$), $W_1 = 1$, -10% variation on $K_{\dot{\theta}}$.

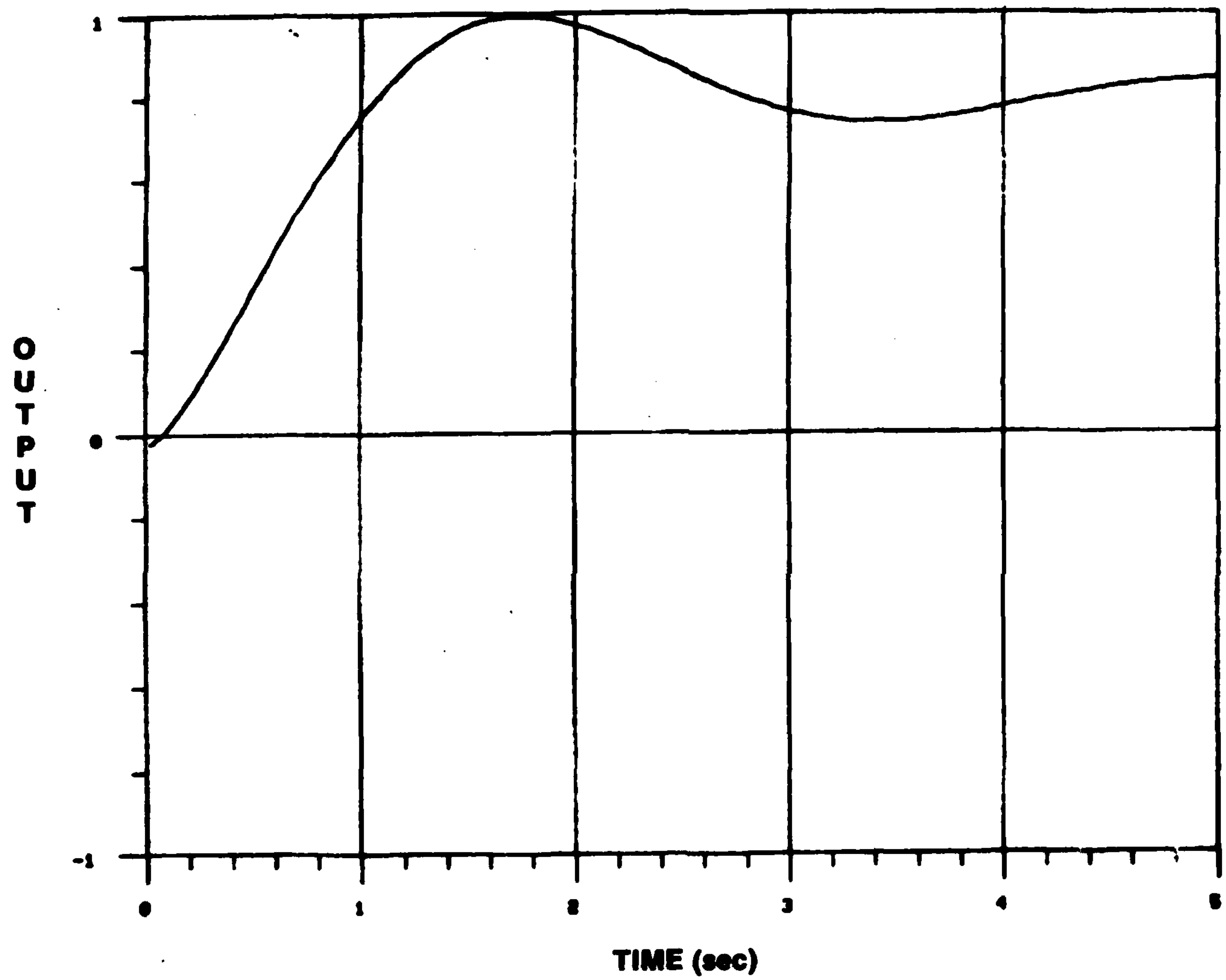


Figure 13. System output response (y), $W_1 = 1$, +10% variation on γ.

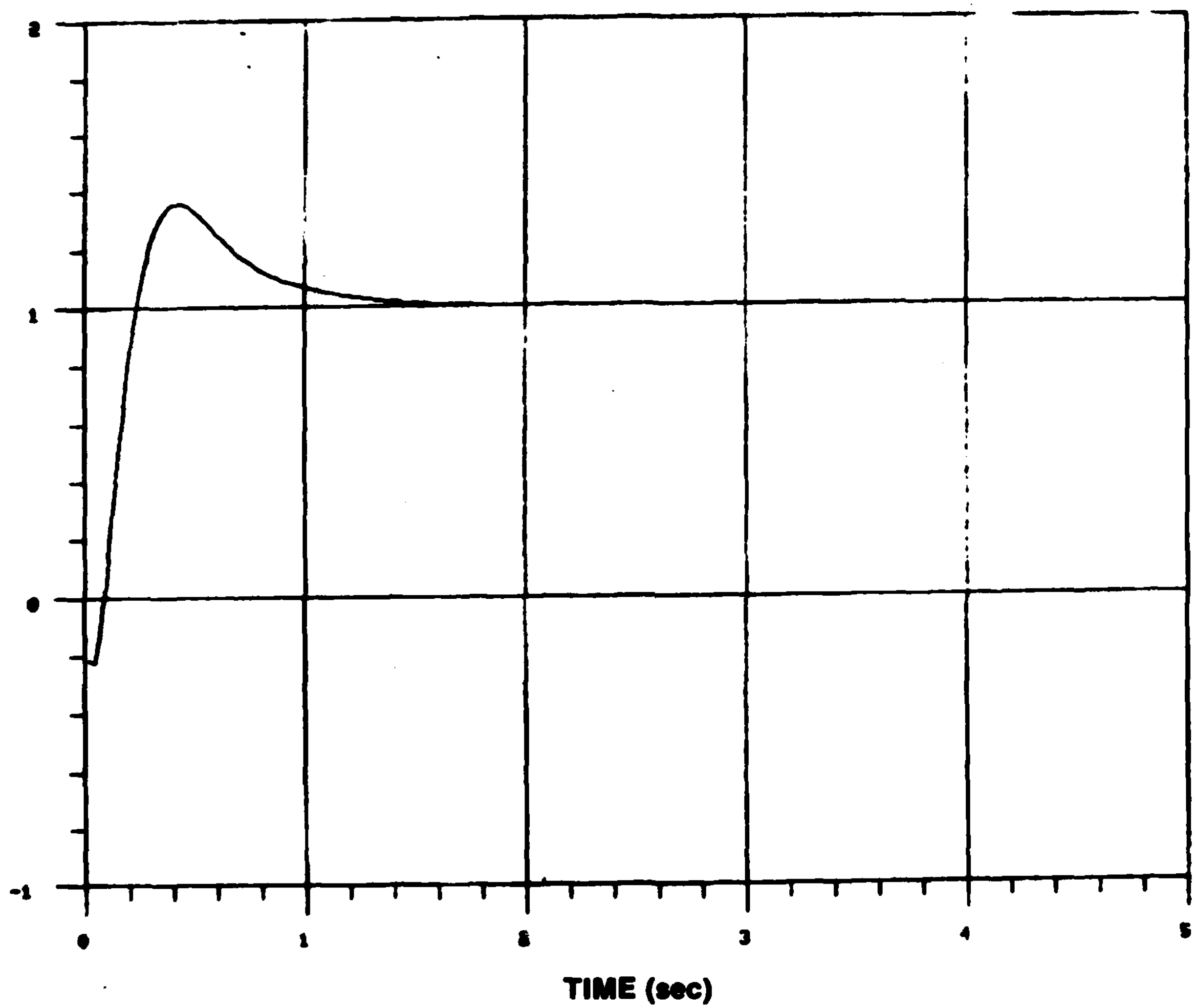


Figure 14. DAC disturbance estimate $(\hat{z}_1)$, $W_1 = 1$,+10% variation on γ.

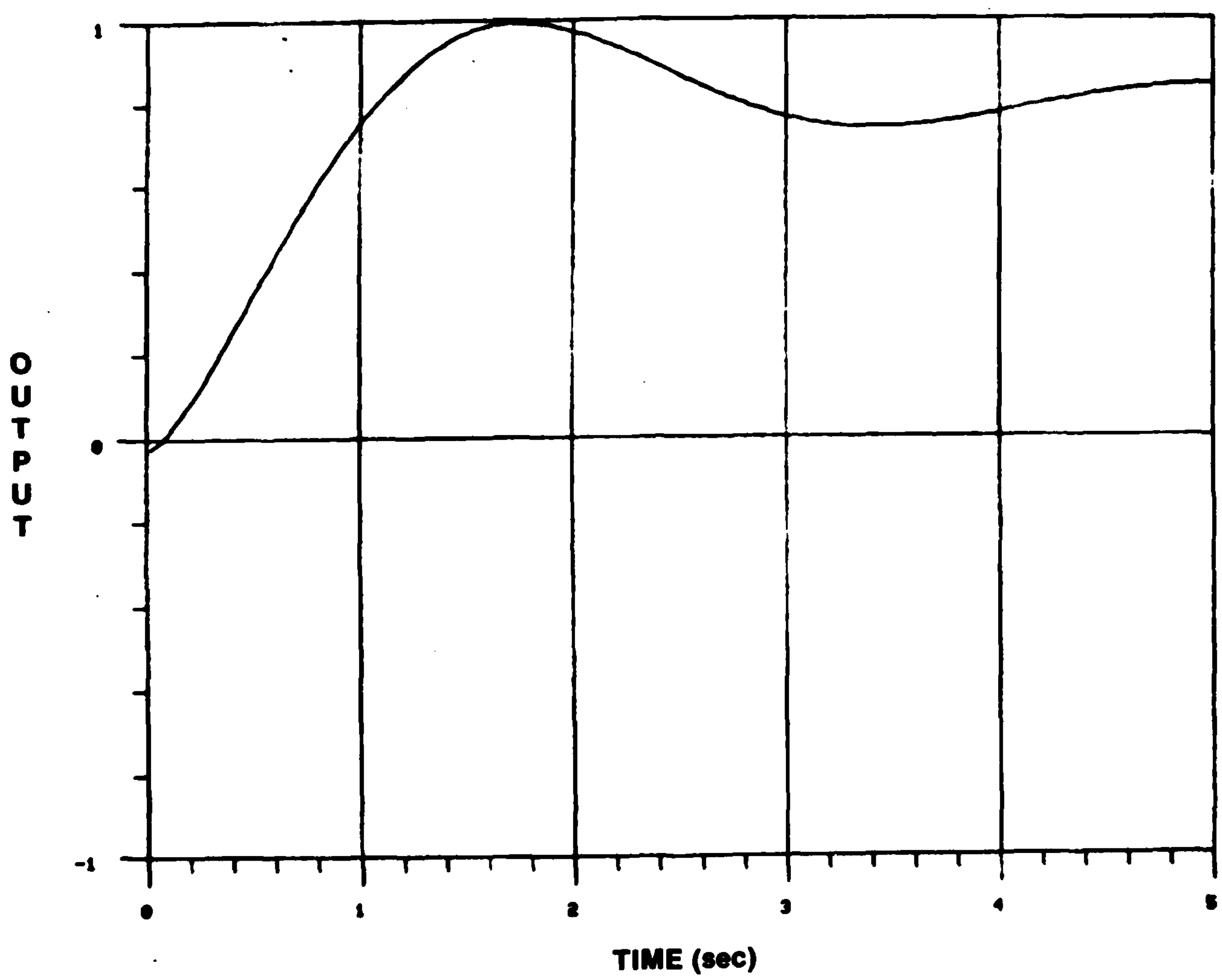


Figure 15. System output response (y), W_1 = 1, -10% variation on γ.

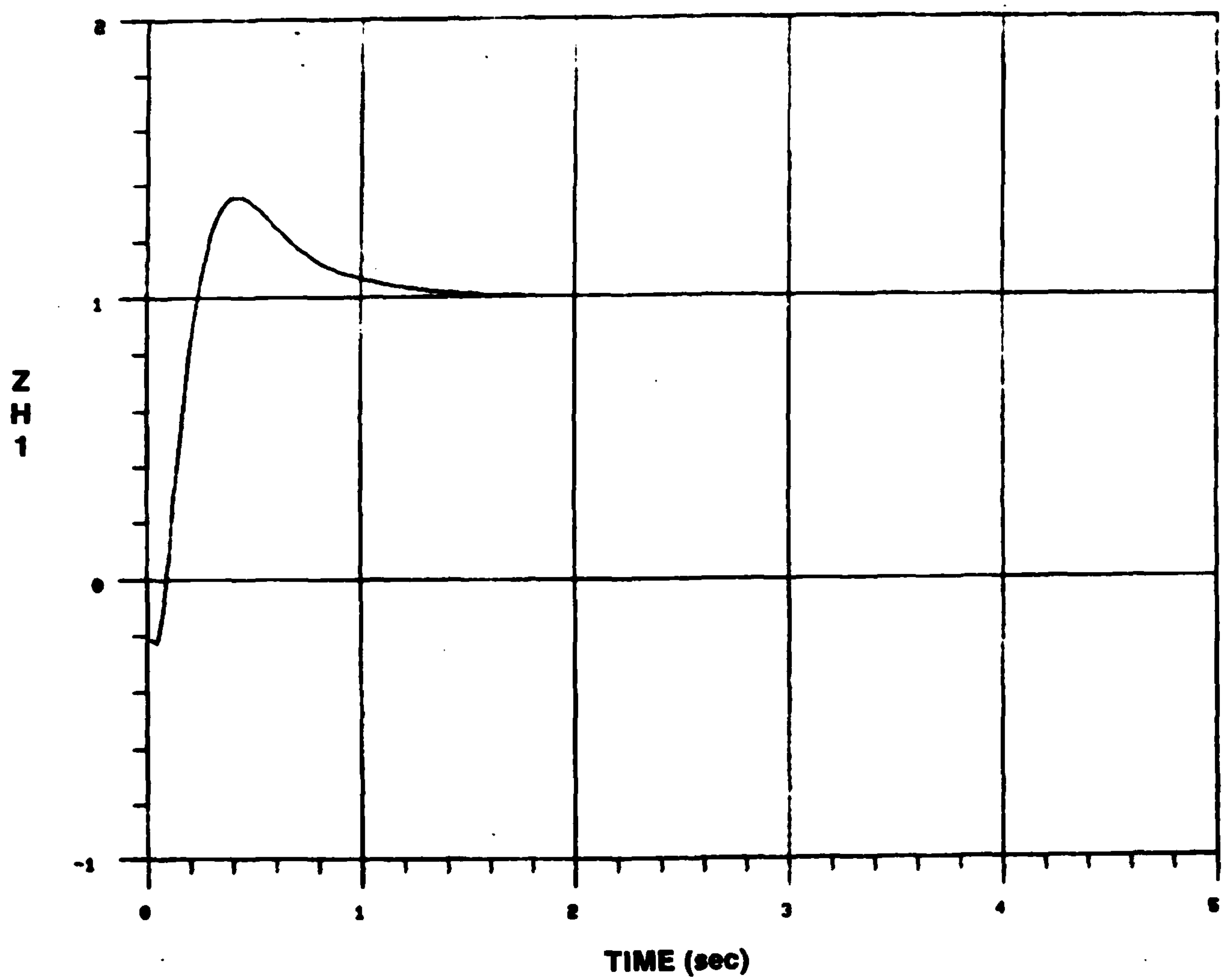


Figure 16. DAC disturbance estimate ($\hat{z}_1$), W_1 = 1, -10% variation on γ.

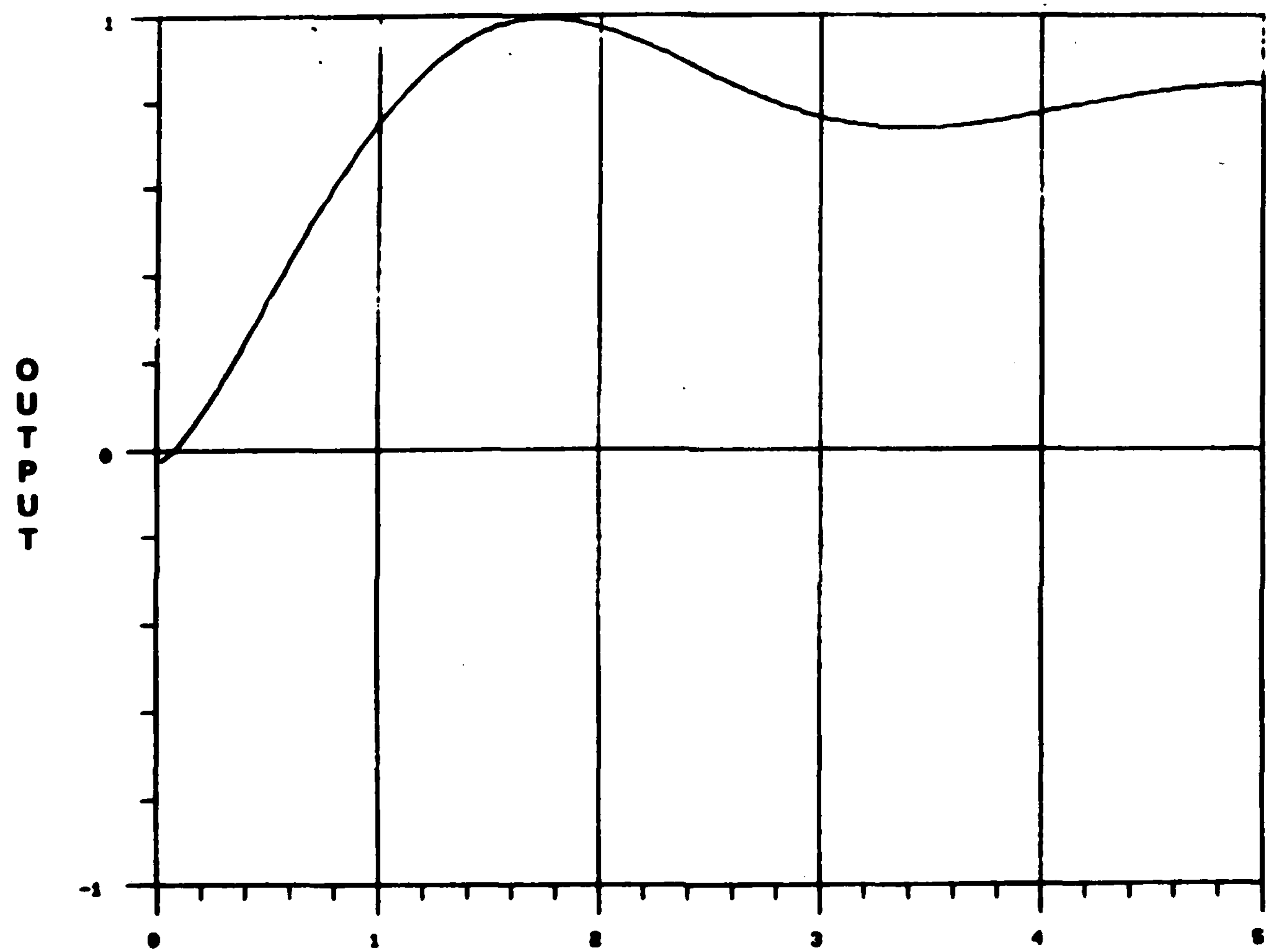


Figure 17. System output response (y), $W_1 = 1$, +10% variation on ζ_A.

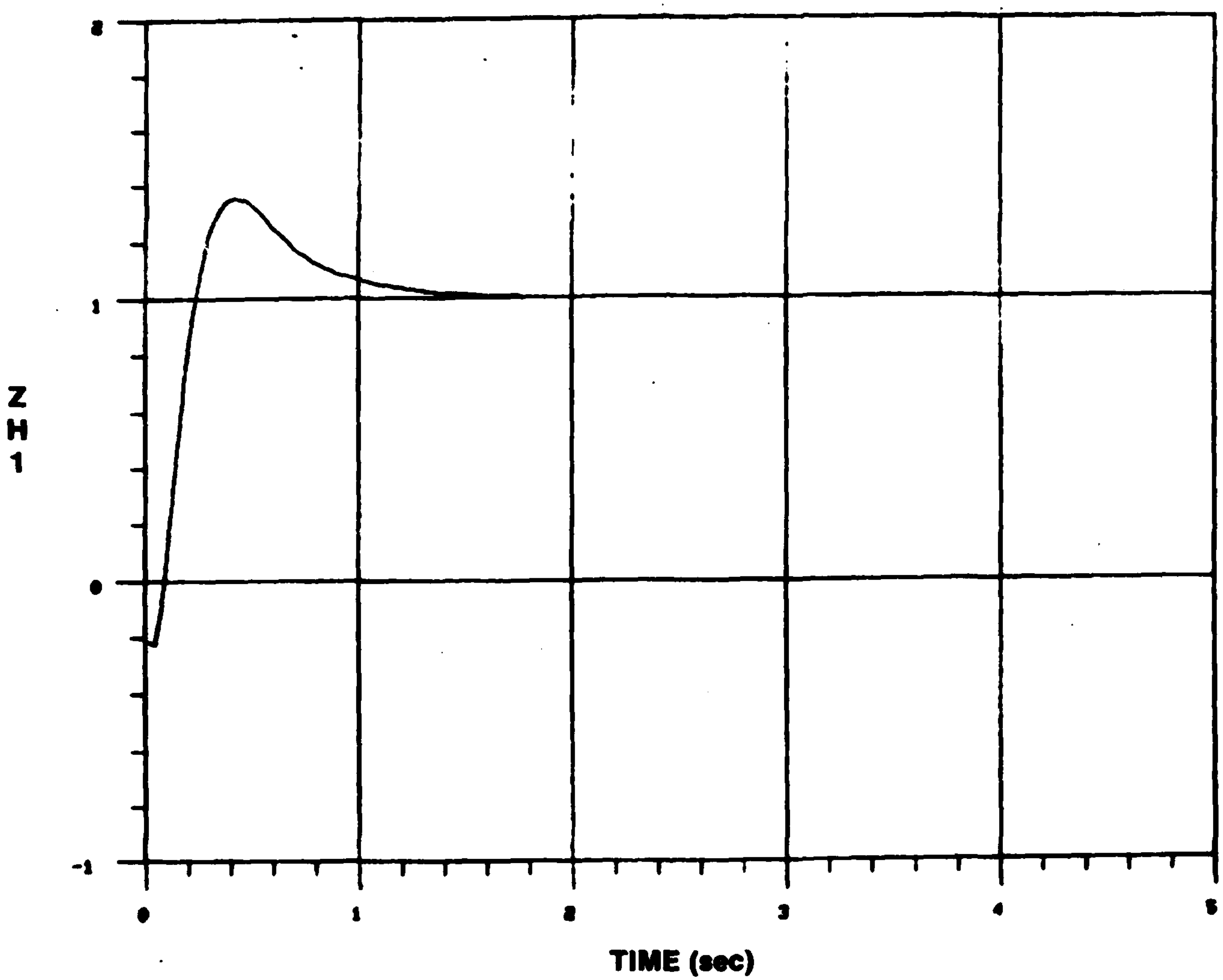


Figure 18. DAC disturbance estimate($\hat{z}_1$), $W_1 = 1$, +10% variation on ζ_A.

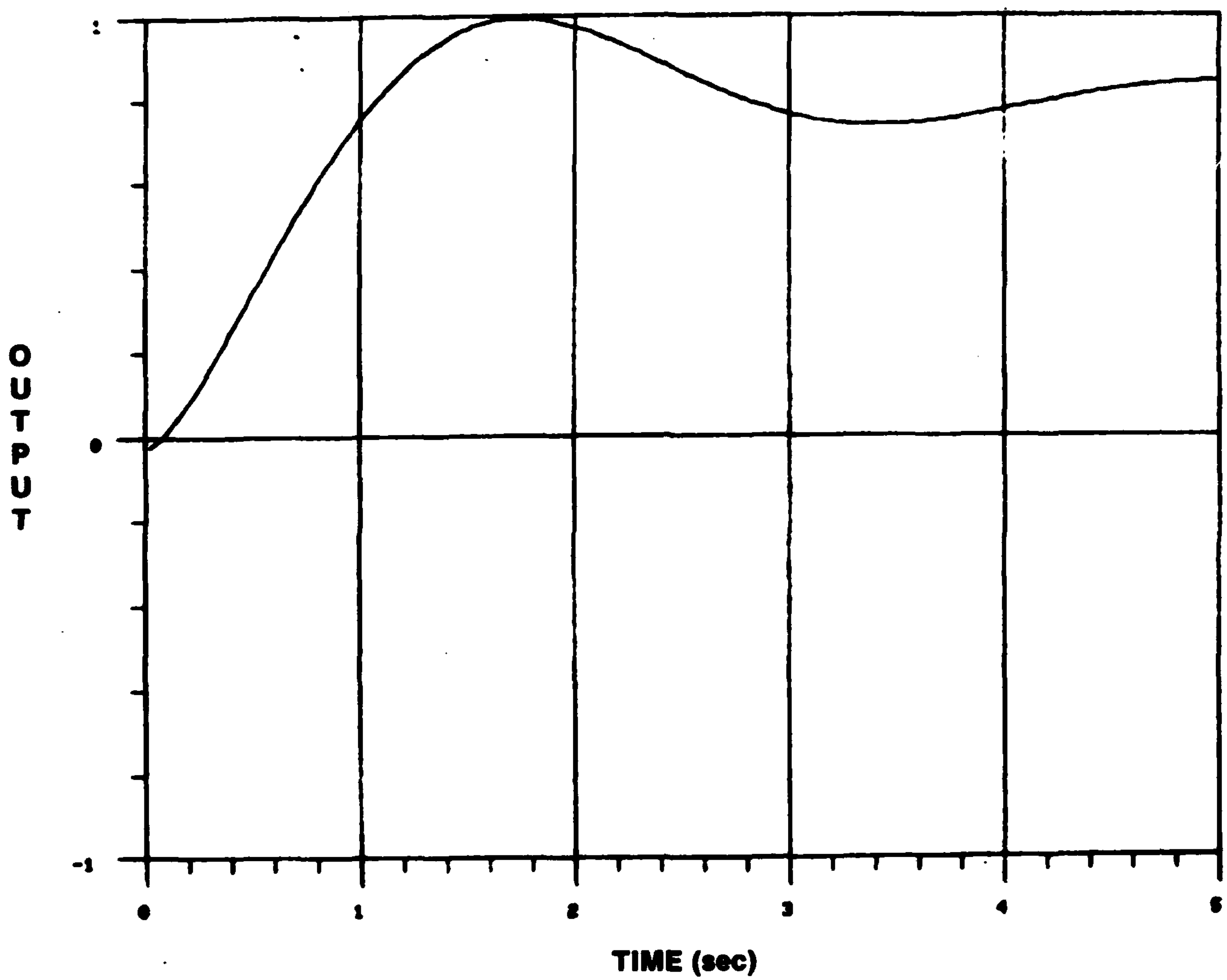


Figure 19. System output response (y), $W_1 = 1$, -10% variation on ζ_A.

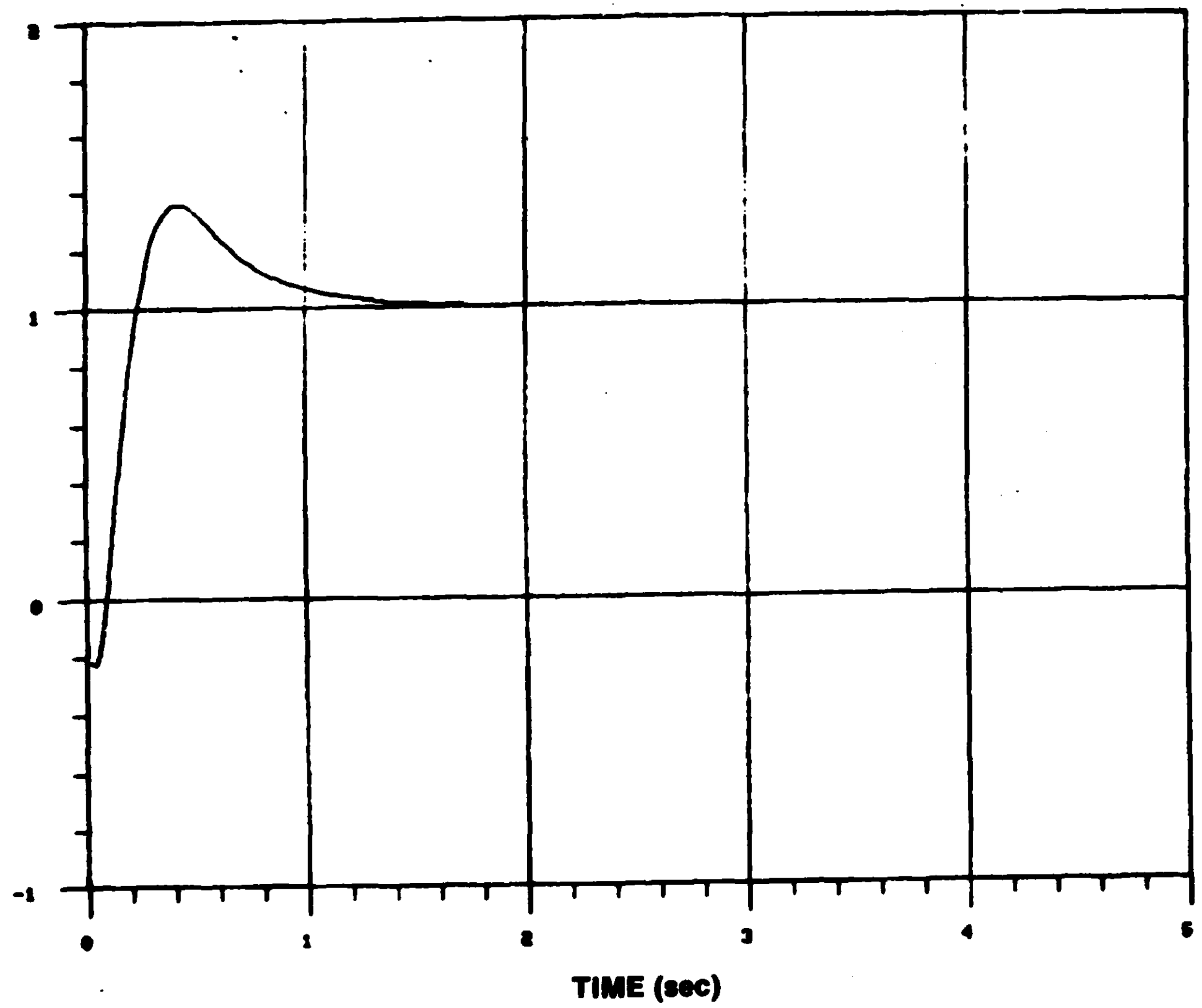


Figure 20. DAC disturbance estimate ($\hat{z}_1$), $W_1 = 1$, -10% variation on ζ_A.

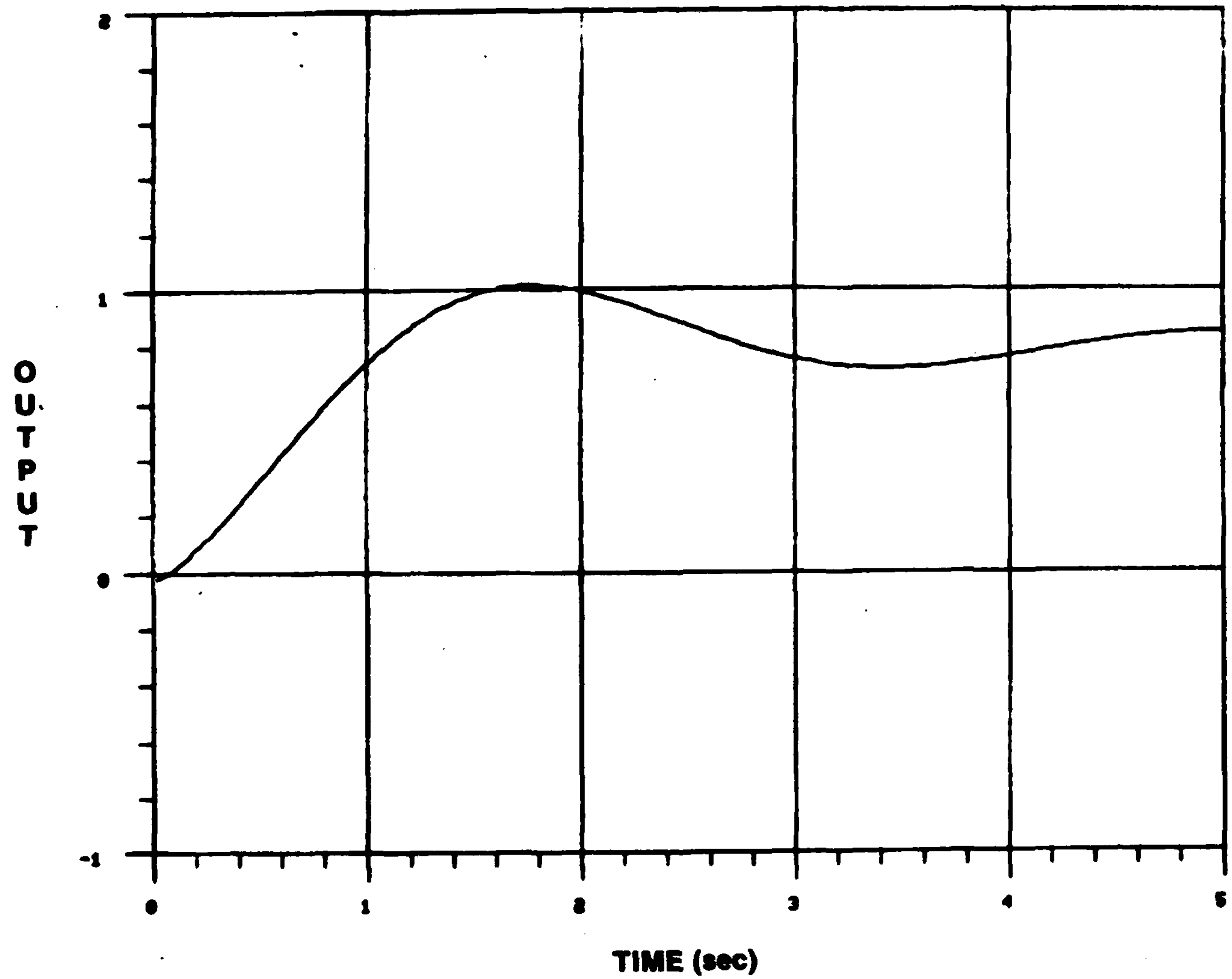


Figure 21. System output response (y), $W_1 = 1$, +10% variation on ω_A.

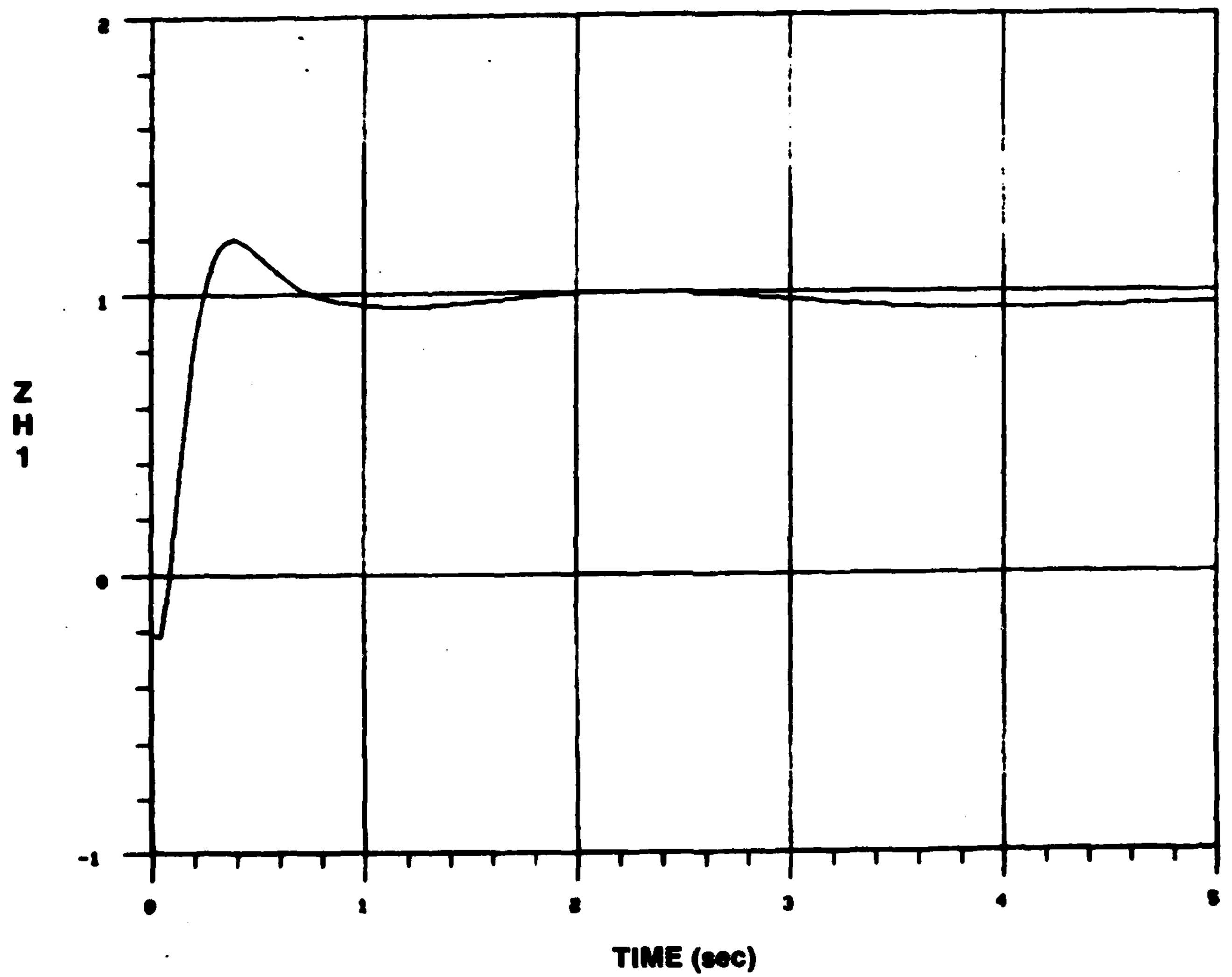


Figure 22. DAC disturbance estimate($\hat{z}_1$), $W_1 = 1$, +10% variation on ω_A.

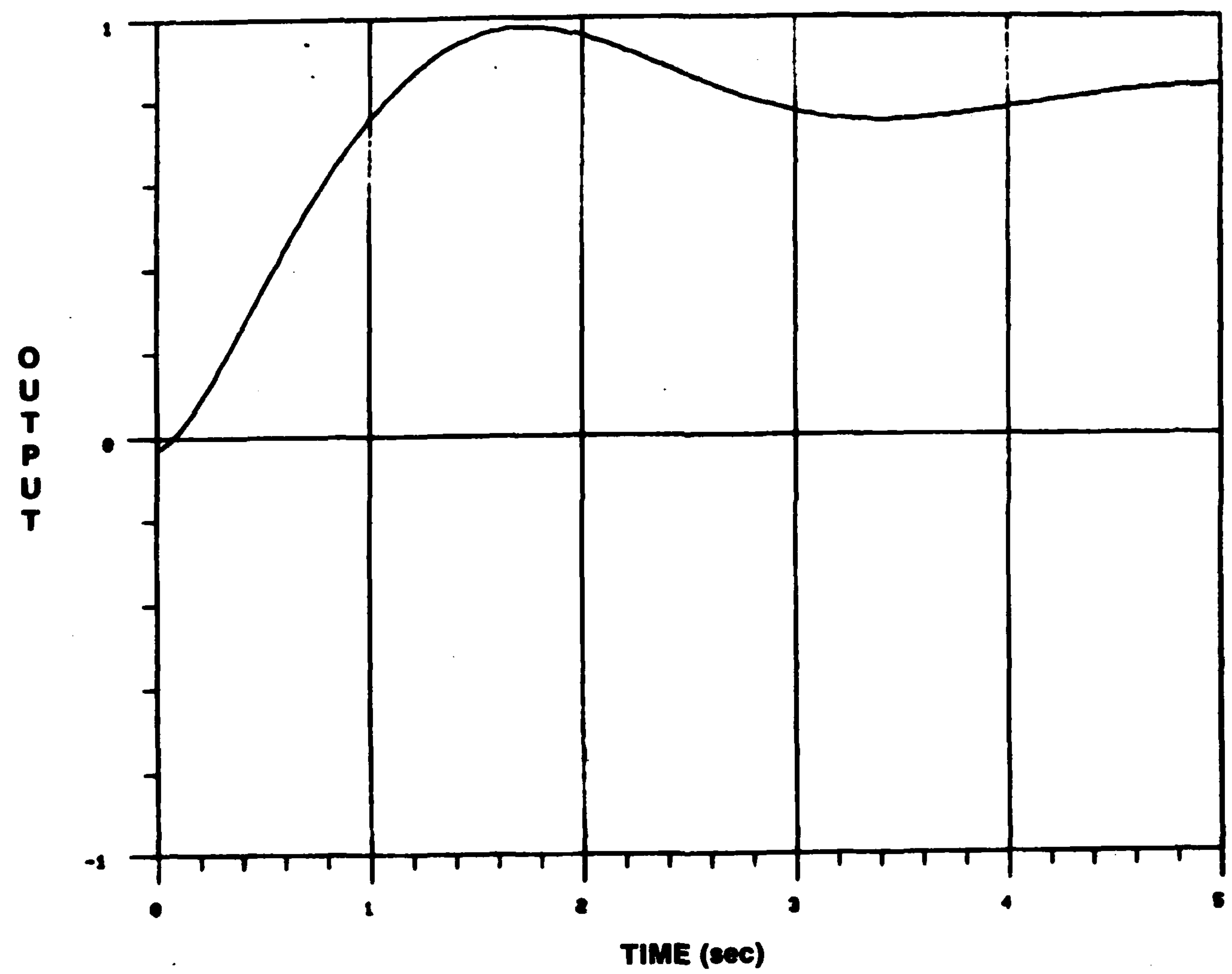


Figure 23. System output response (y), $W_1 = 1$, -10% variation on ω_A.

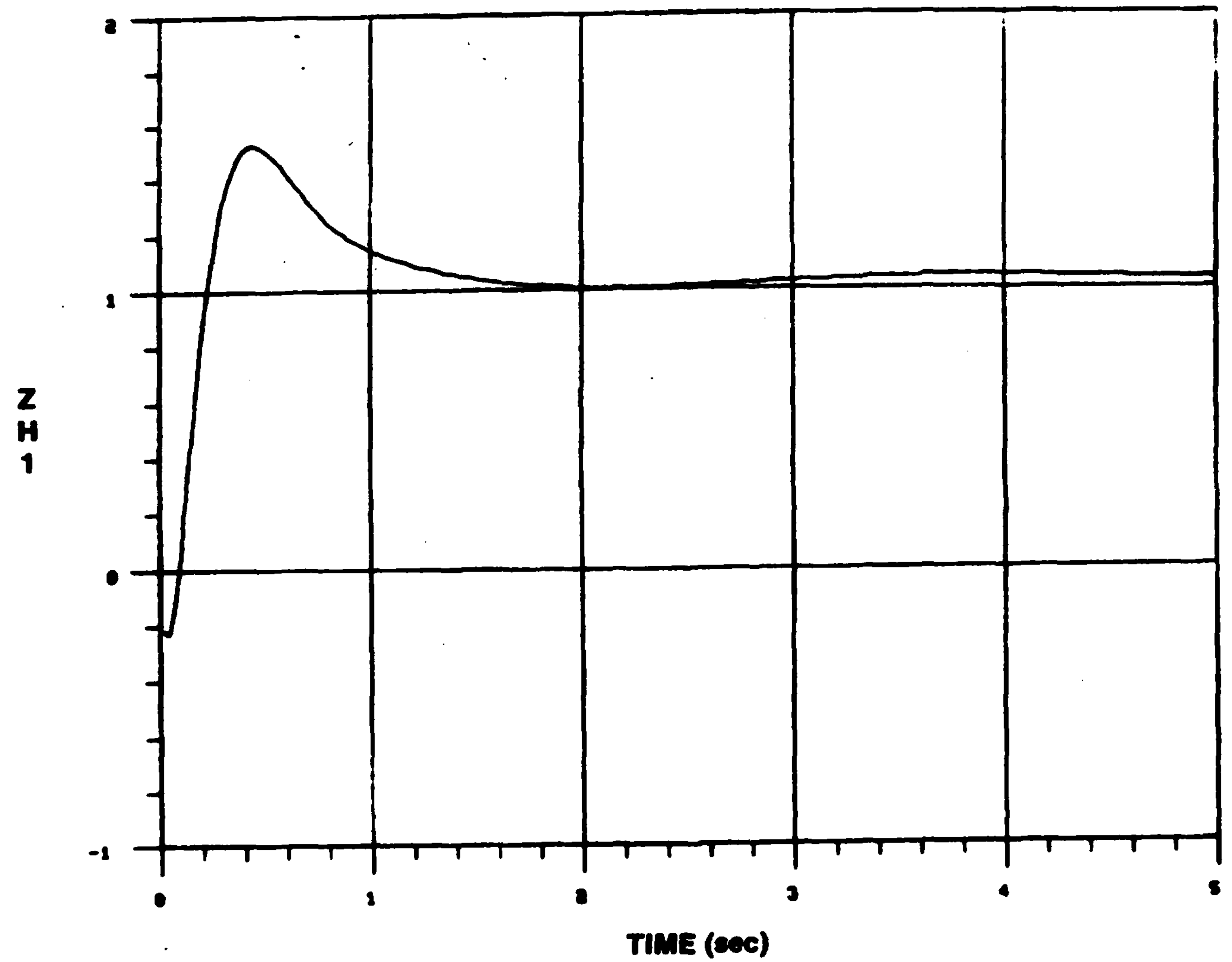


Figure 24. DAC disturbance estimate($\hat{z}_1$), $W_1 = 1$, -10% variation on ω_A.

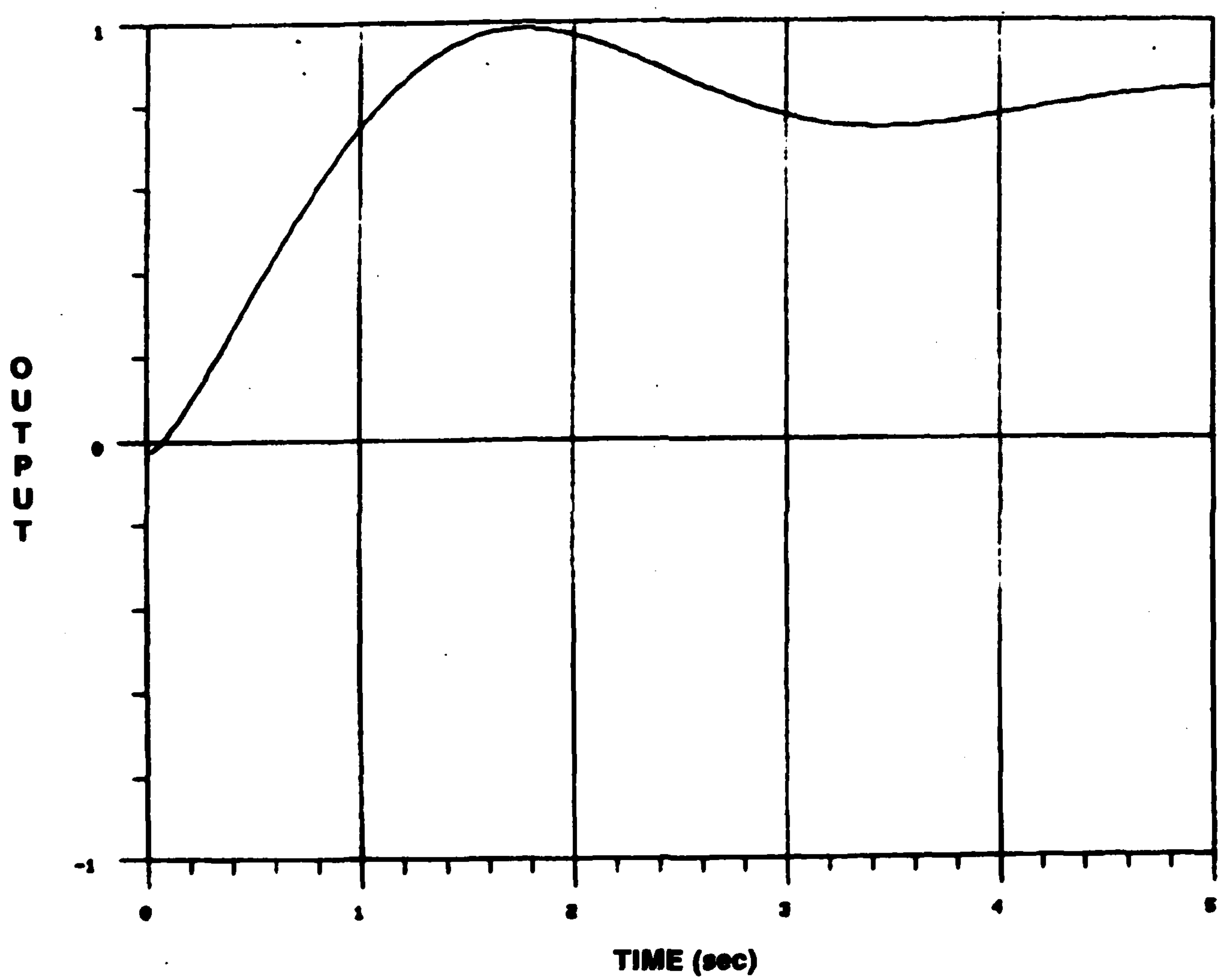


Figure 25. System output response (y), $W_1 = 1$, +10% variation on ω_f^2.

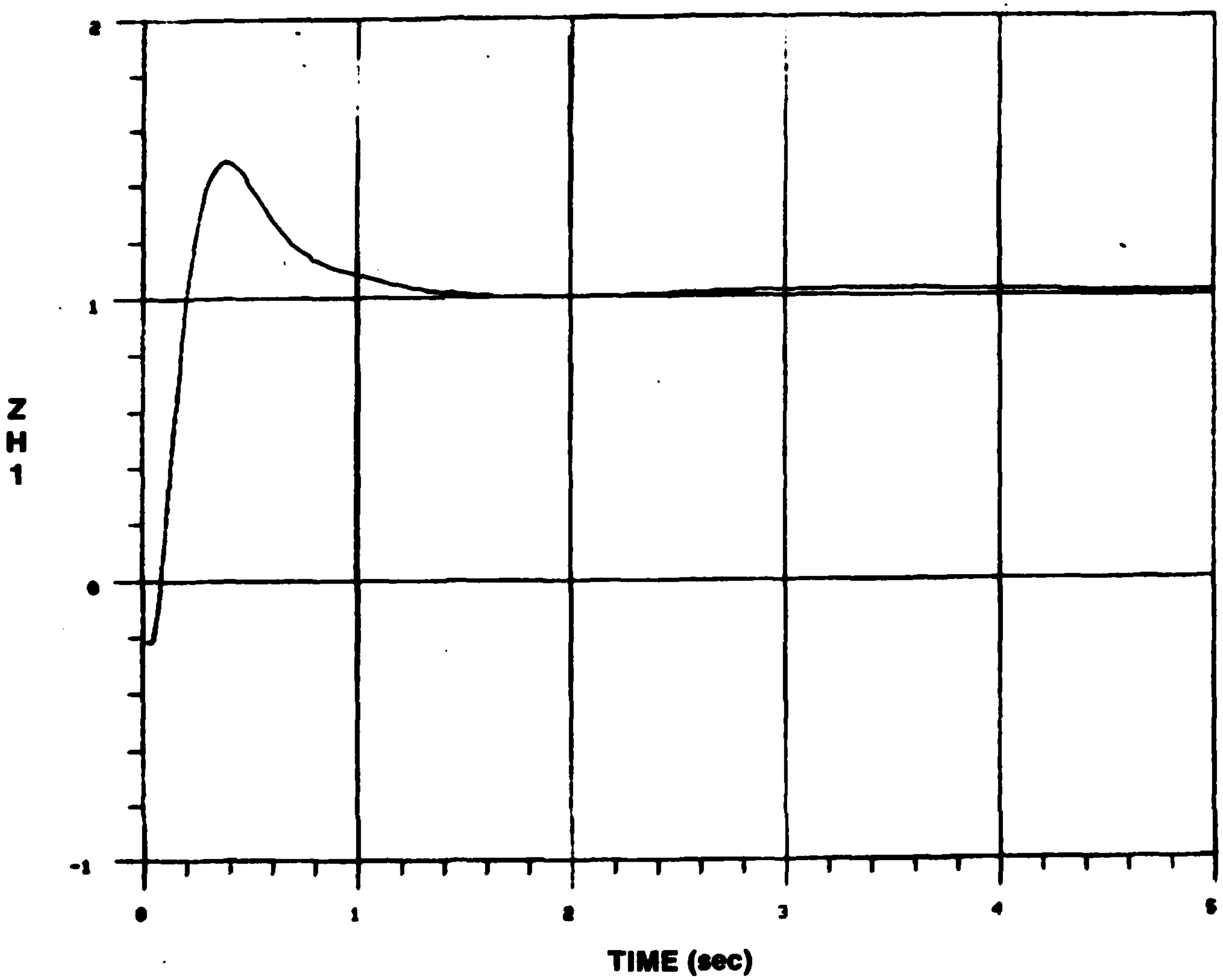


Figure 26. DAC disturbance estimate $(\hat{z}_1)$, $W_1 = 1$, +10% variation on ω_f^2.

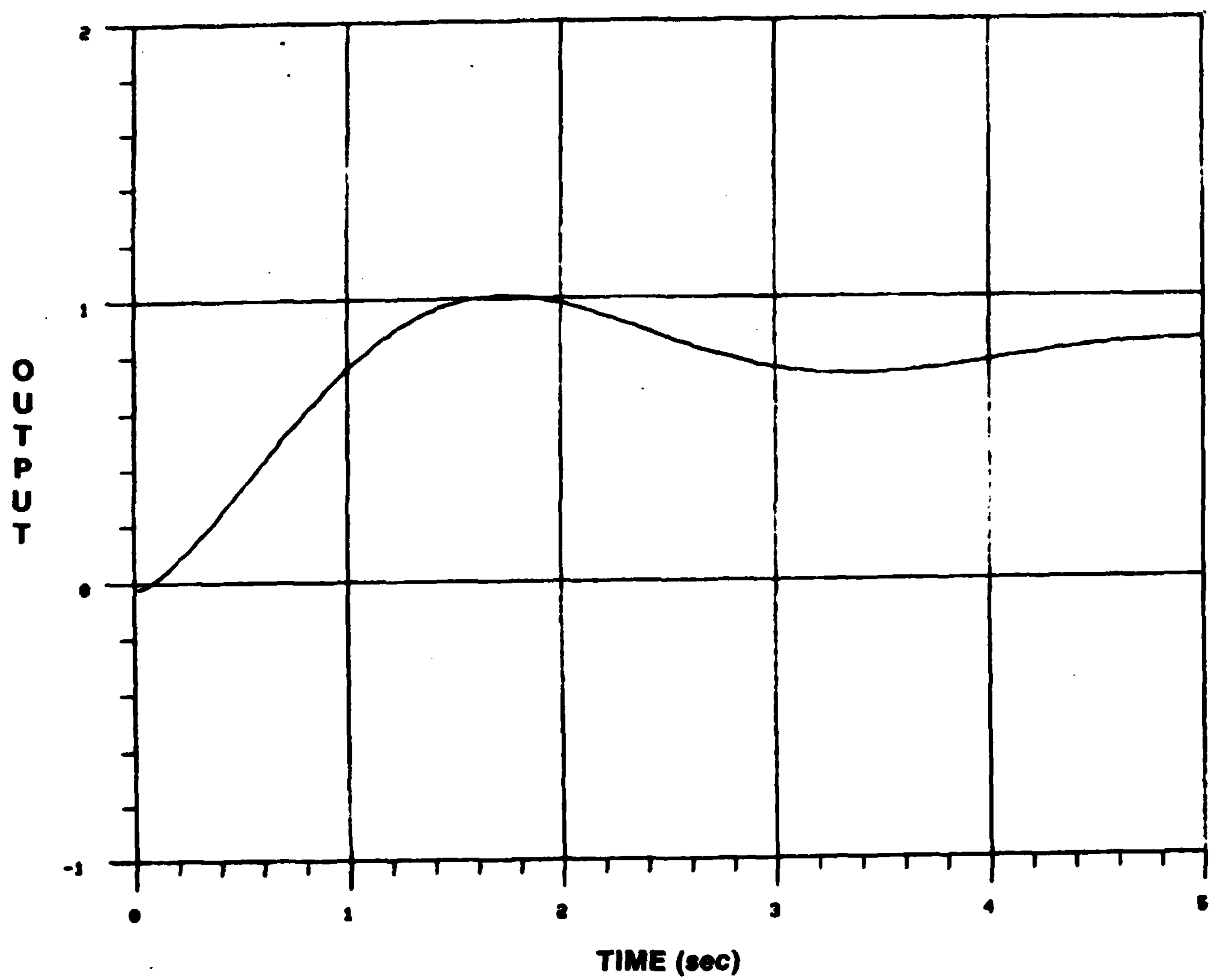


Figure 27. System output response (y), W_1 = 1, -10% variation on ω_f^2.

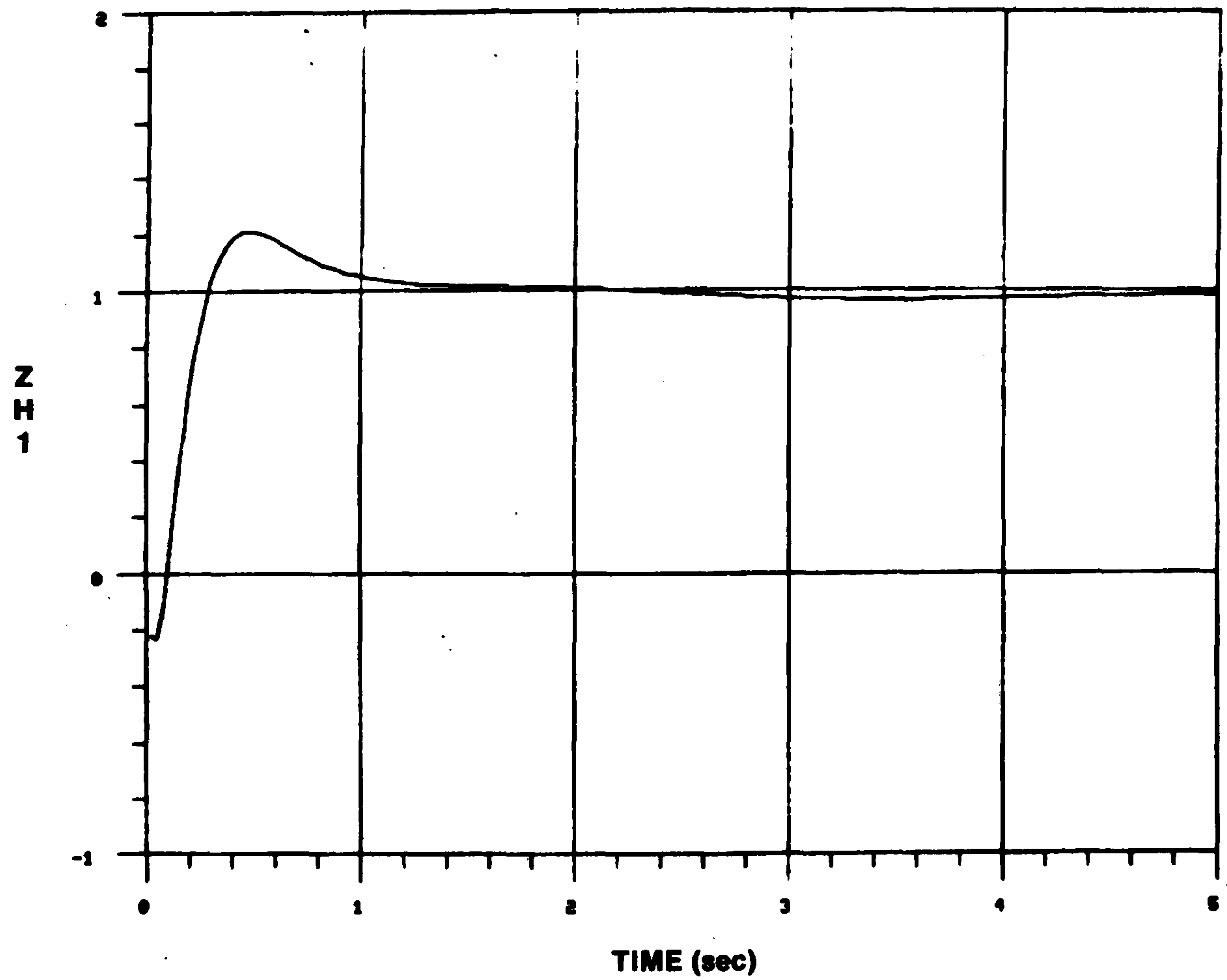


Figure 28. DAC disturbance estimate $(\hat{z}_1)$, $W_1 = 1$, -10% variation on ω_η^2.

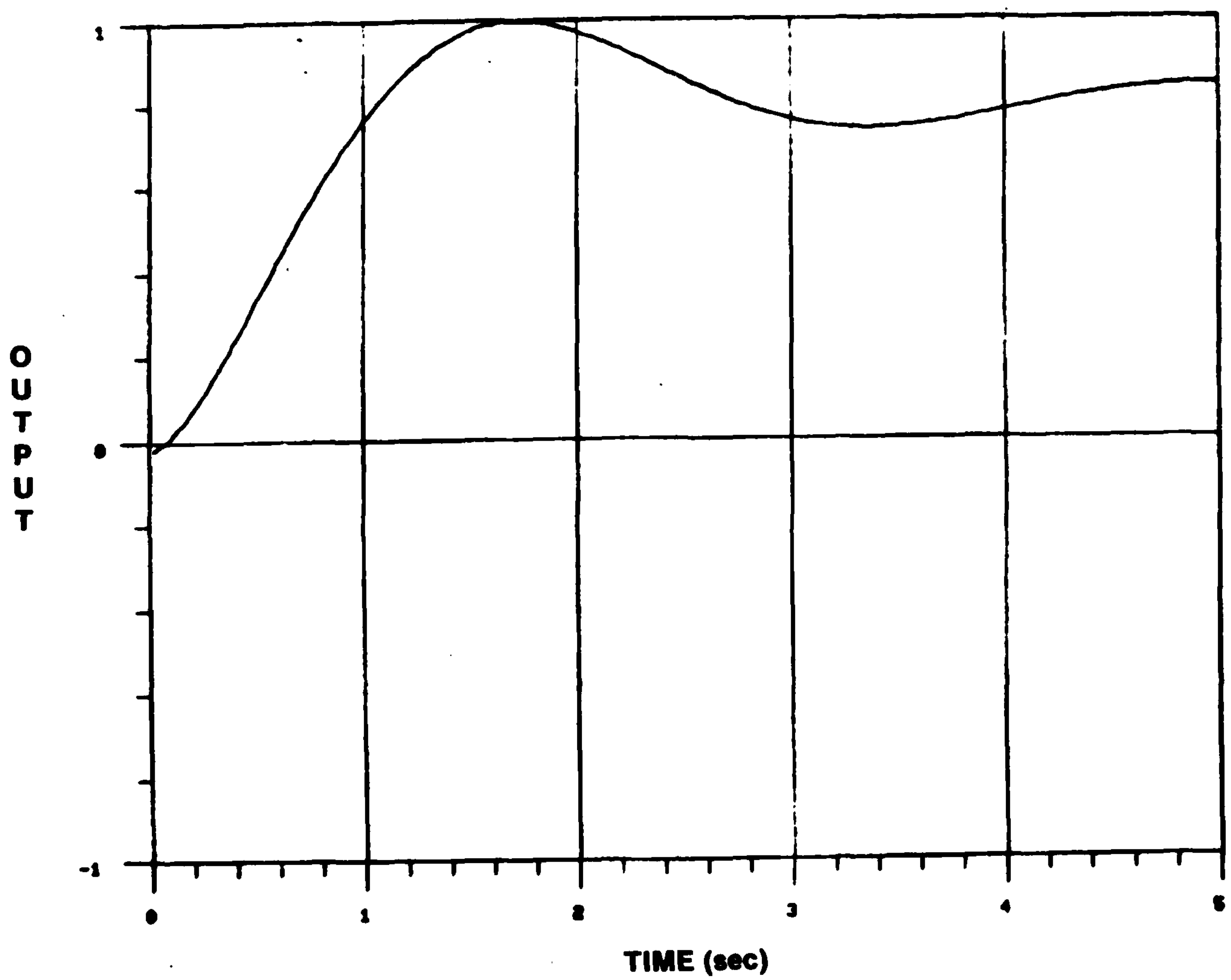


Figure 29. System output response (y), $W_1 = 1$, +10% variation on C_R.

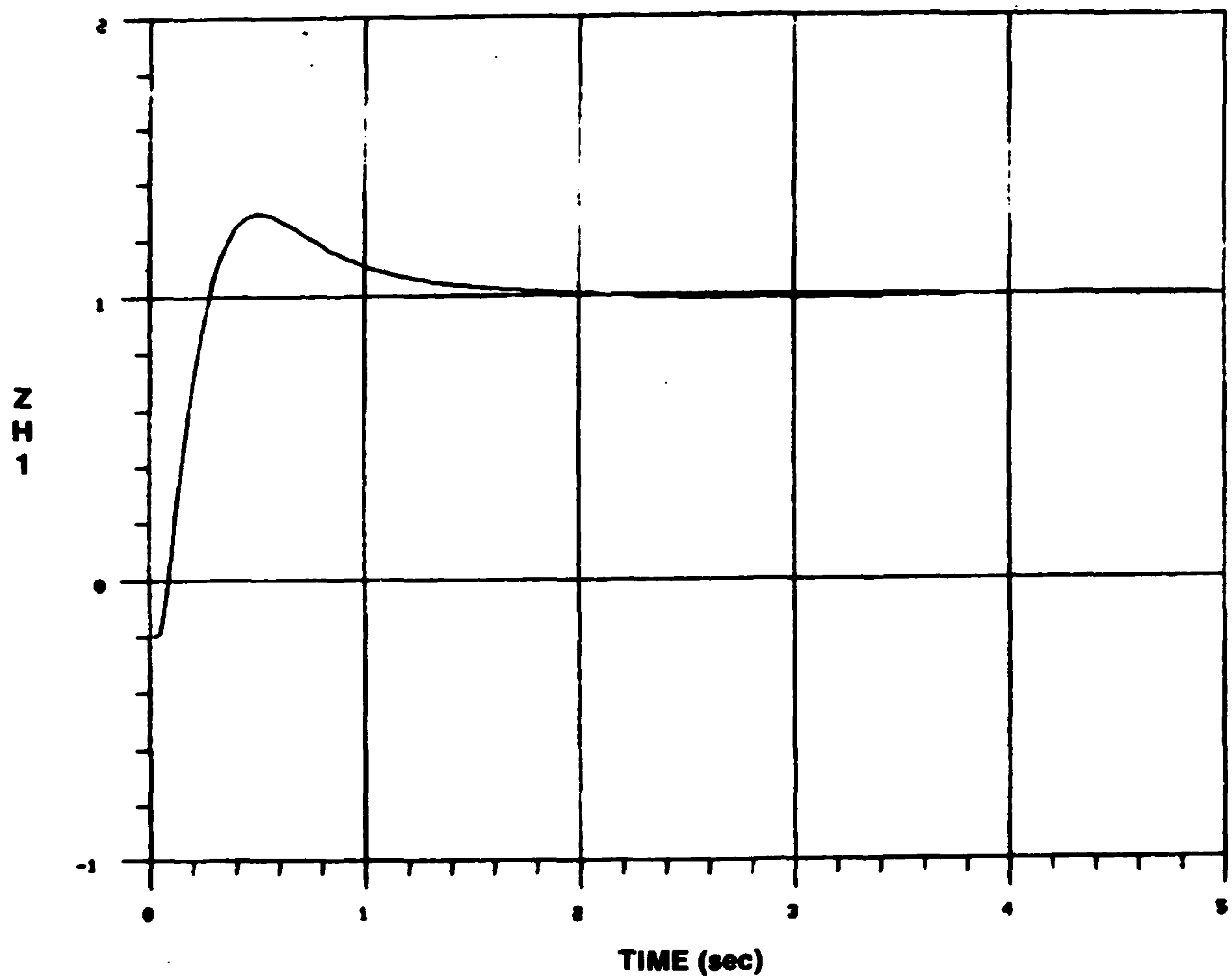


Figure 30. DAC disturbance estimate($\hat{z}_1$), $W_1 = 1$, +10% variation on C_R.

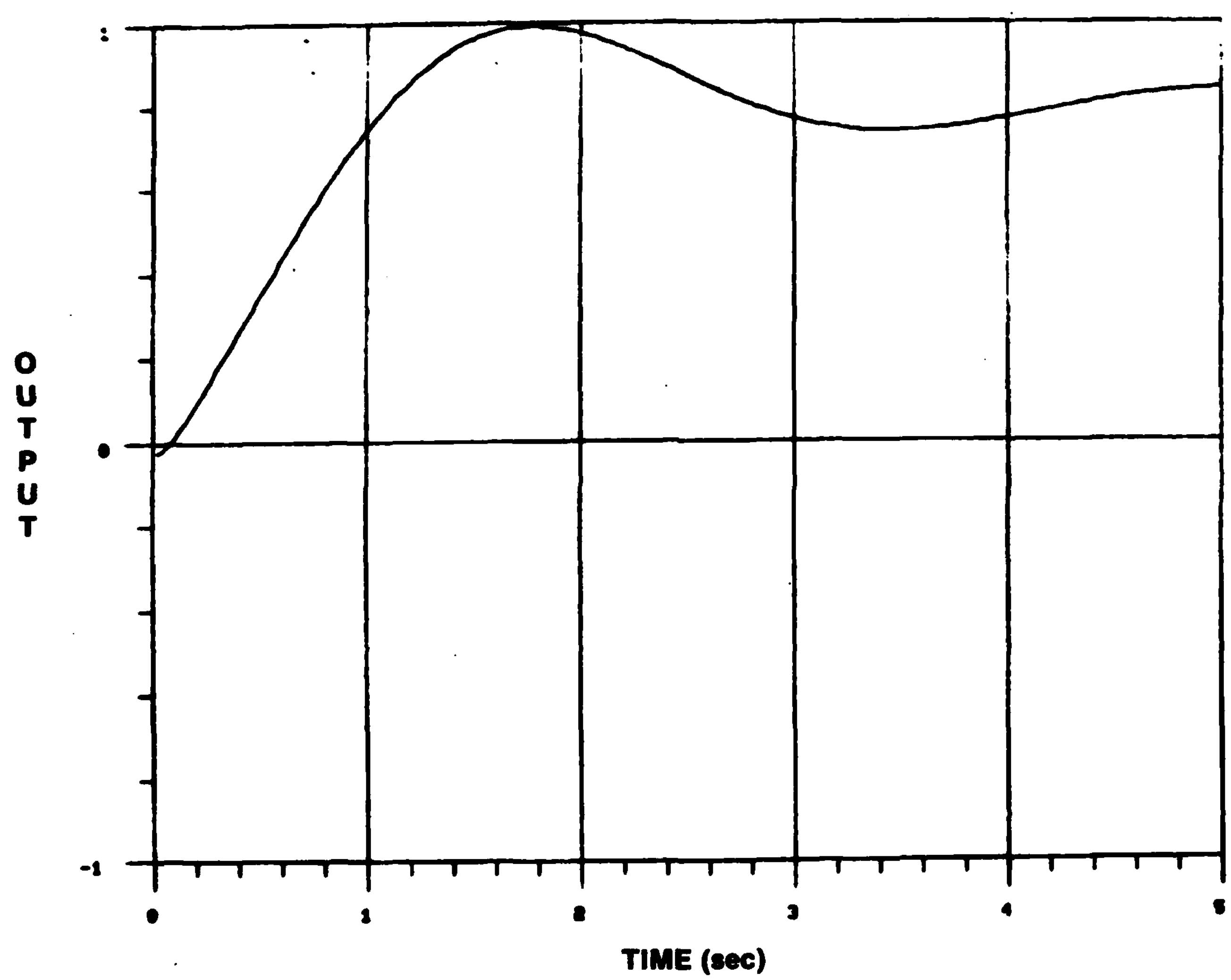


Figure 31. System output response (y), $W_1 = 1$, -10% variation on C_R.

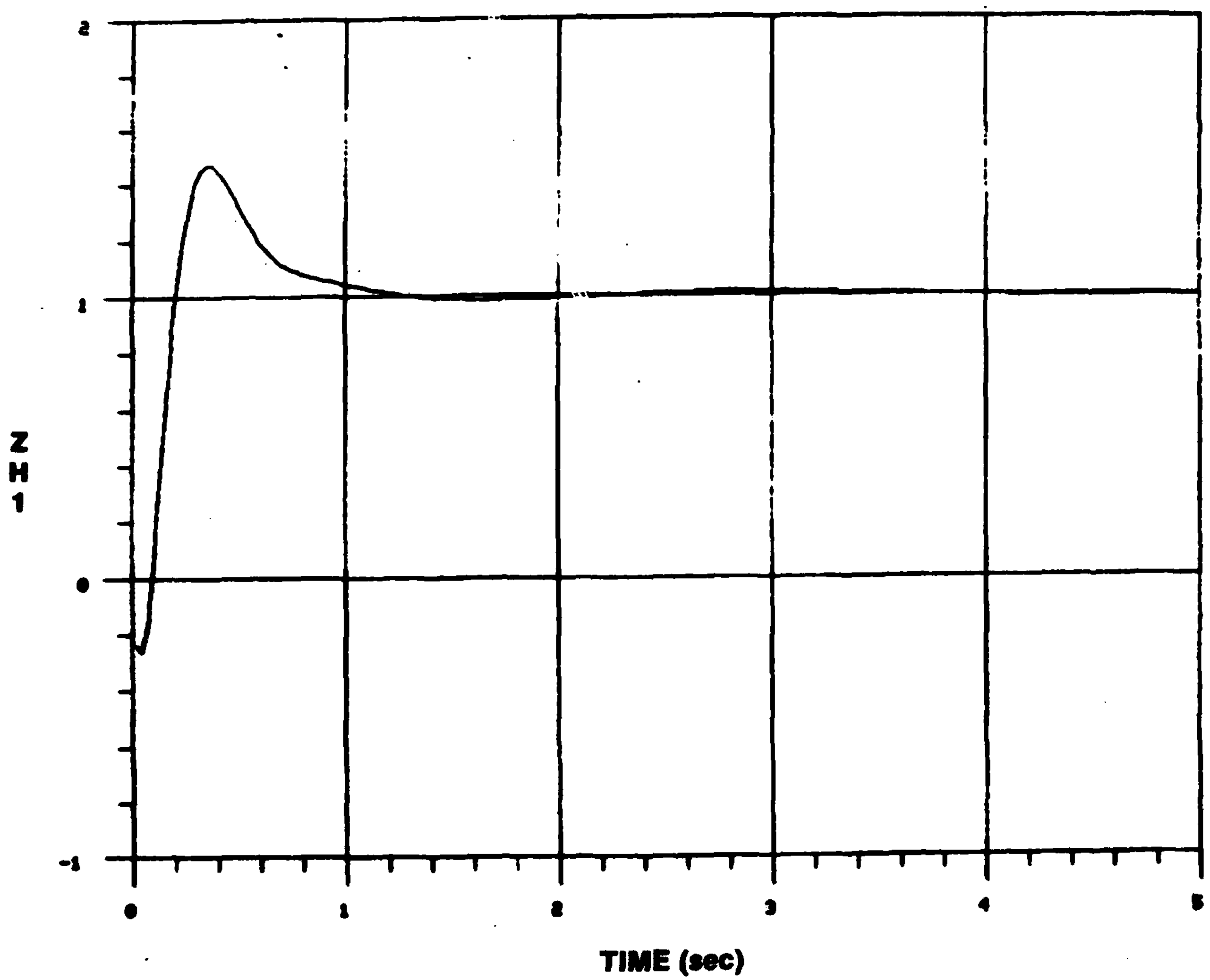


Figure 32. DAC disturbance estimate ($\hat{z}_1$), $W_1 = 1$, -10% variation on C_R.

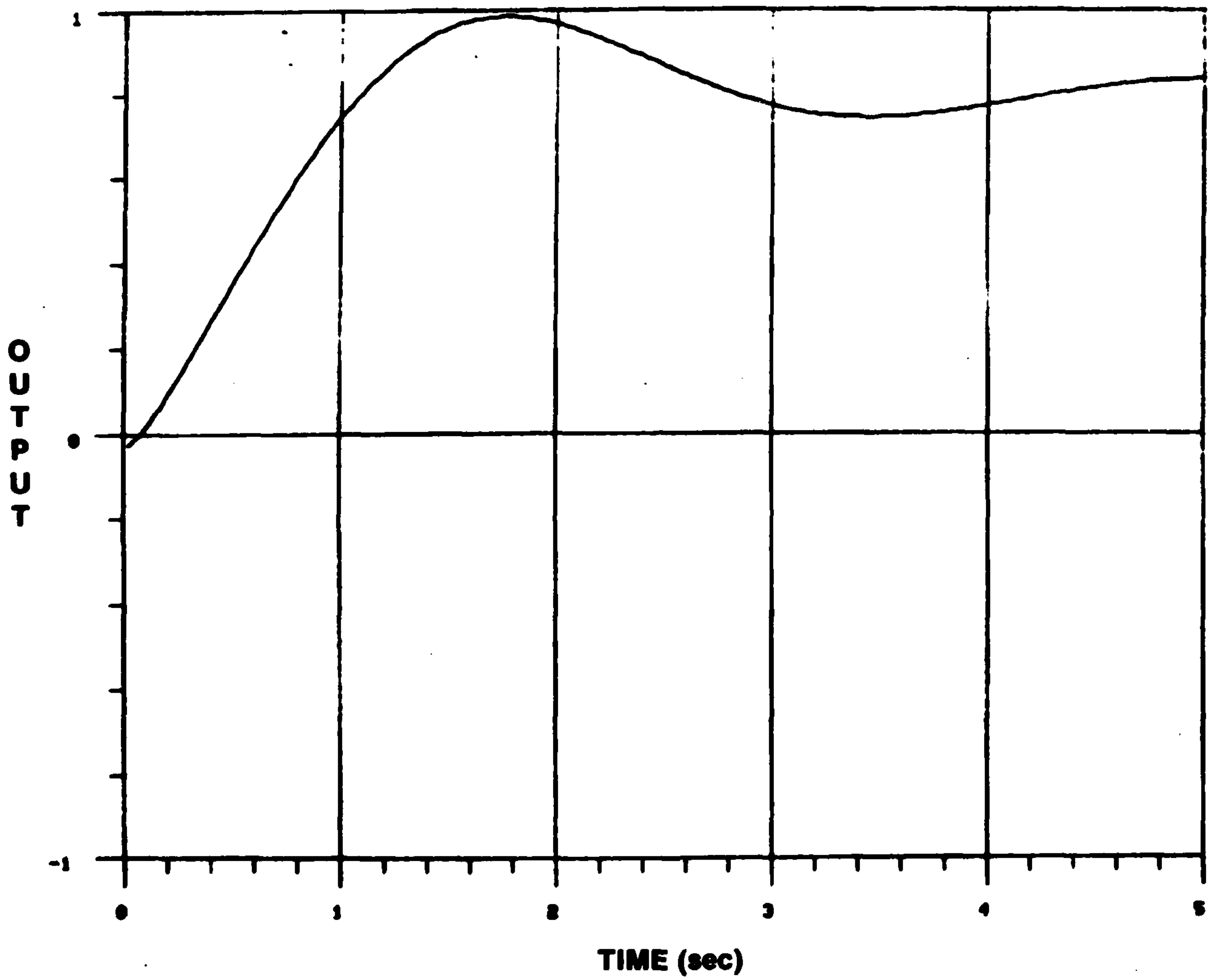


Figure 33. System output response (y), $W_1 = 1$, +10% variation on K_η.

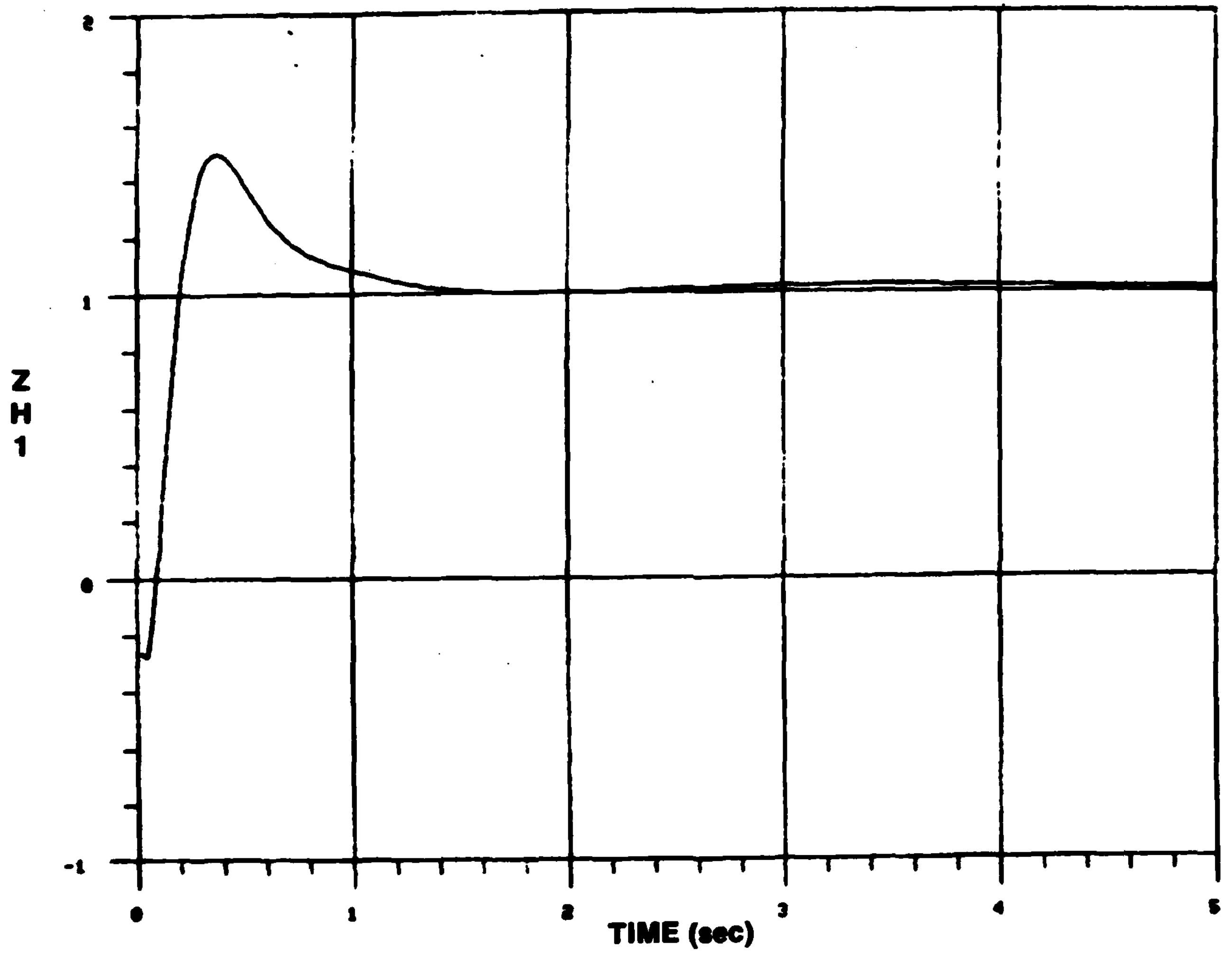


Figure 34. DAC disturbance estimate($\hat{z}_1$), $W_1 = 1$, +10% variation on K_η.

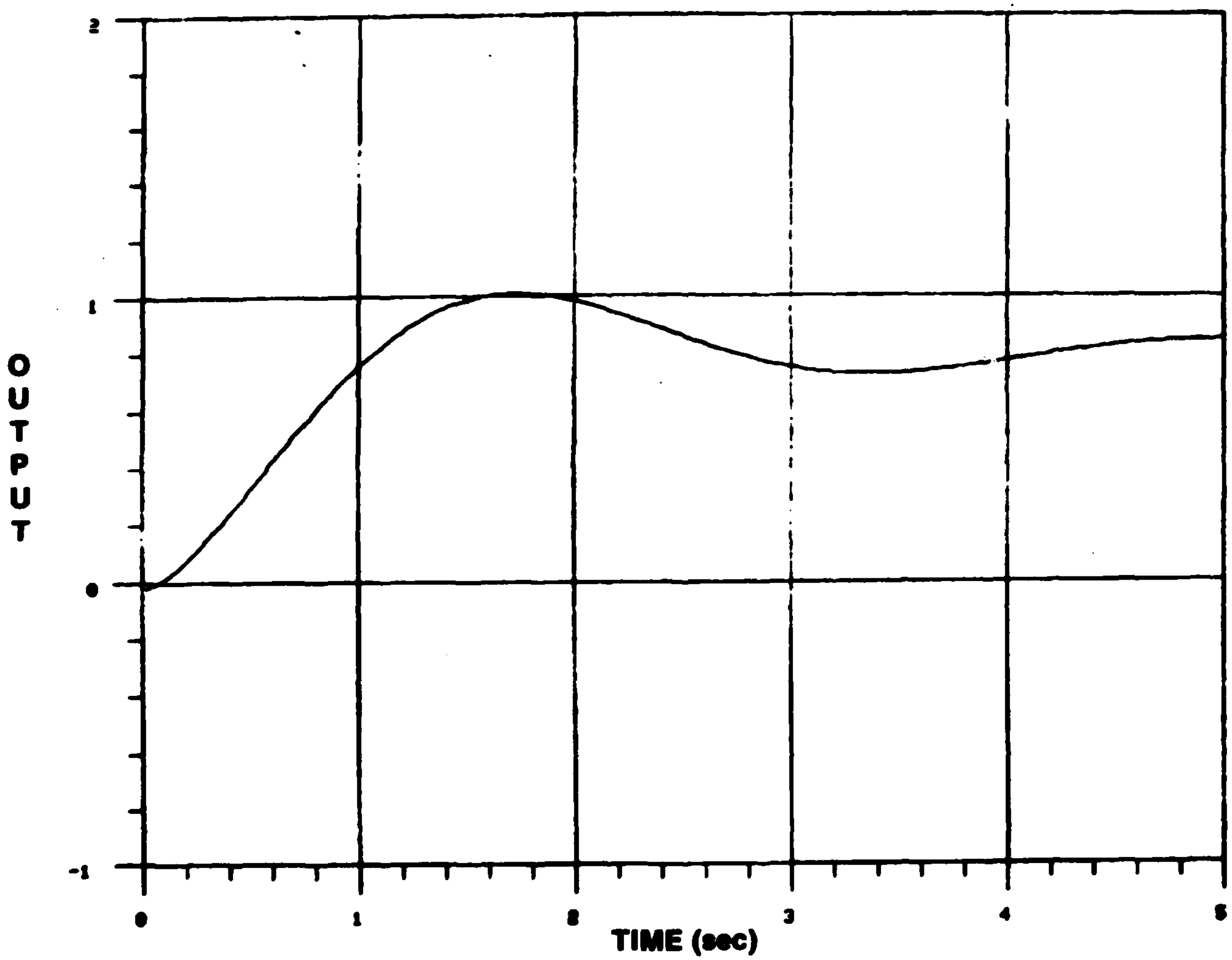


Figure 35. System output response (y), $W_1 = 1$, -10% variation on K_η.

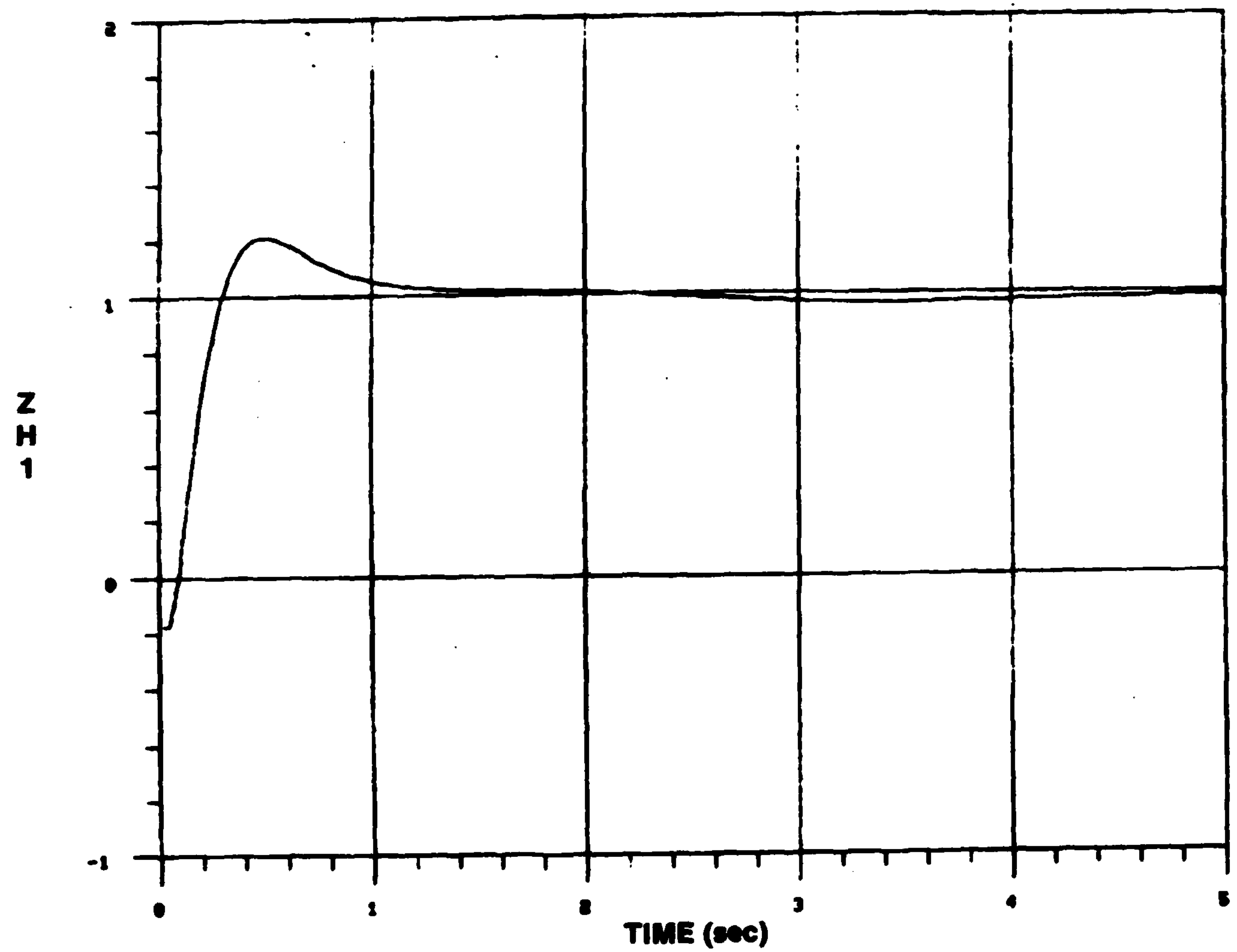


Figure 36. DAC disturbance estimate ($\hat{z}_1$), $W_1 = 1$, -10% variation on K_η.

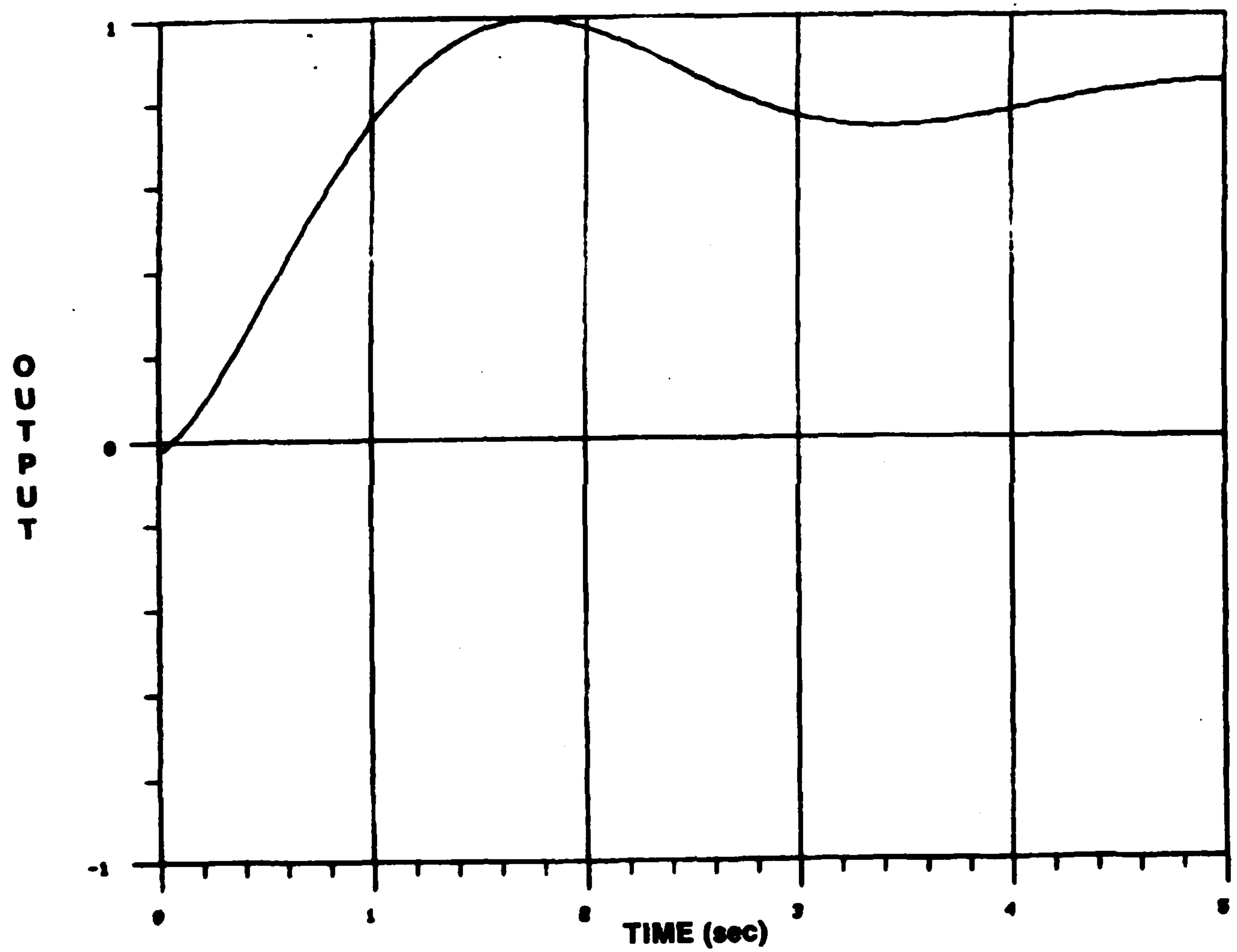


Figure 37. System output response (y), W_1 = 1, +20% variation on all parameters.

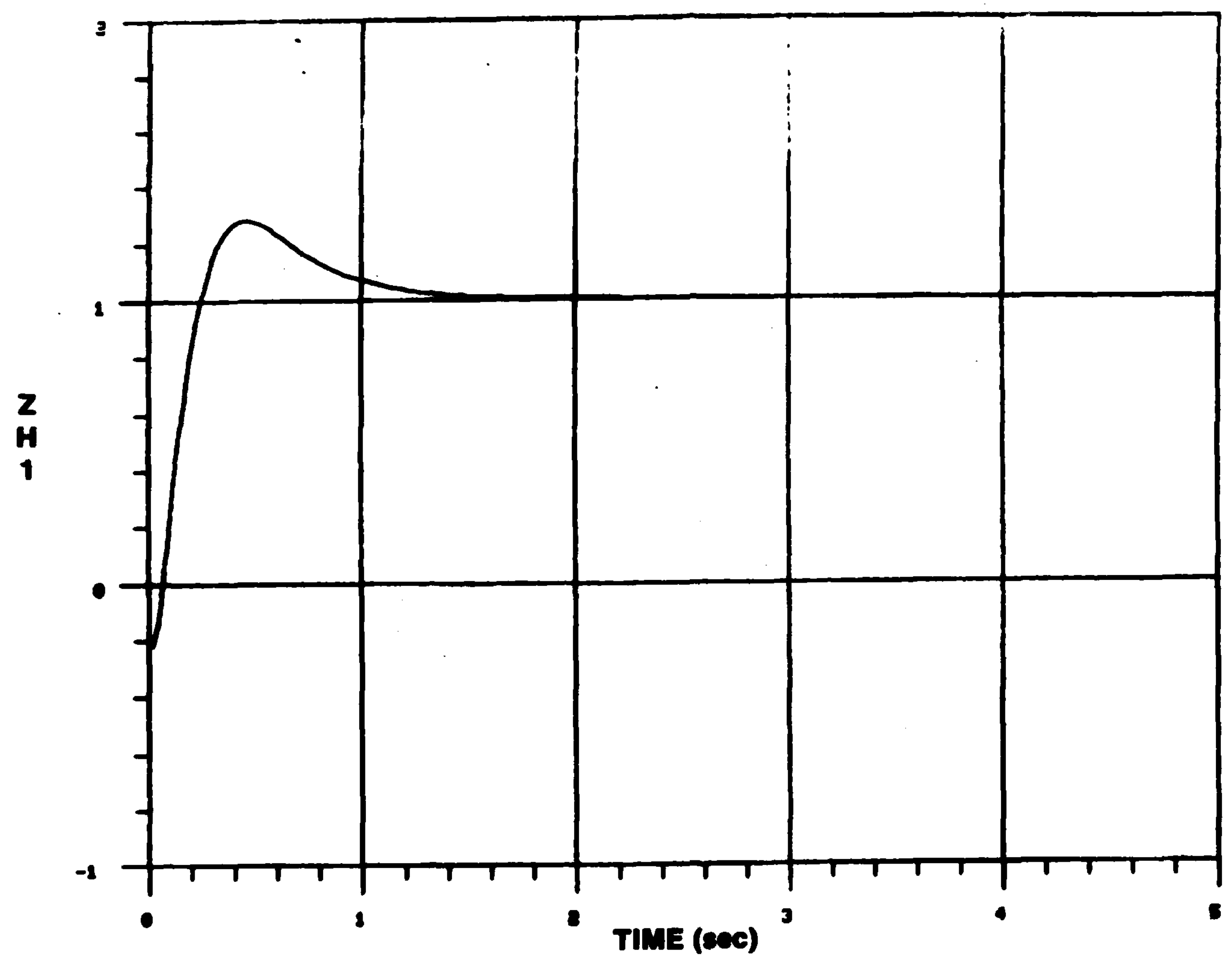


Figure 38. DAC disturbance estimate ($\hat{z}_1$), $W_1 = 1$, +20% variation on all parameters.

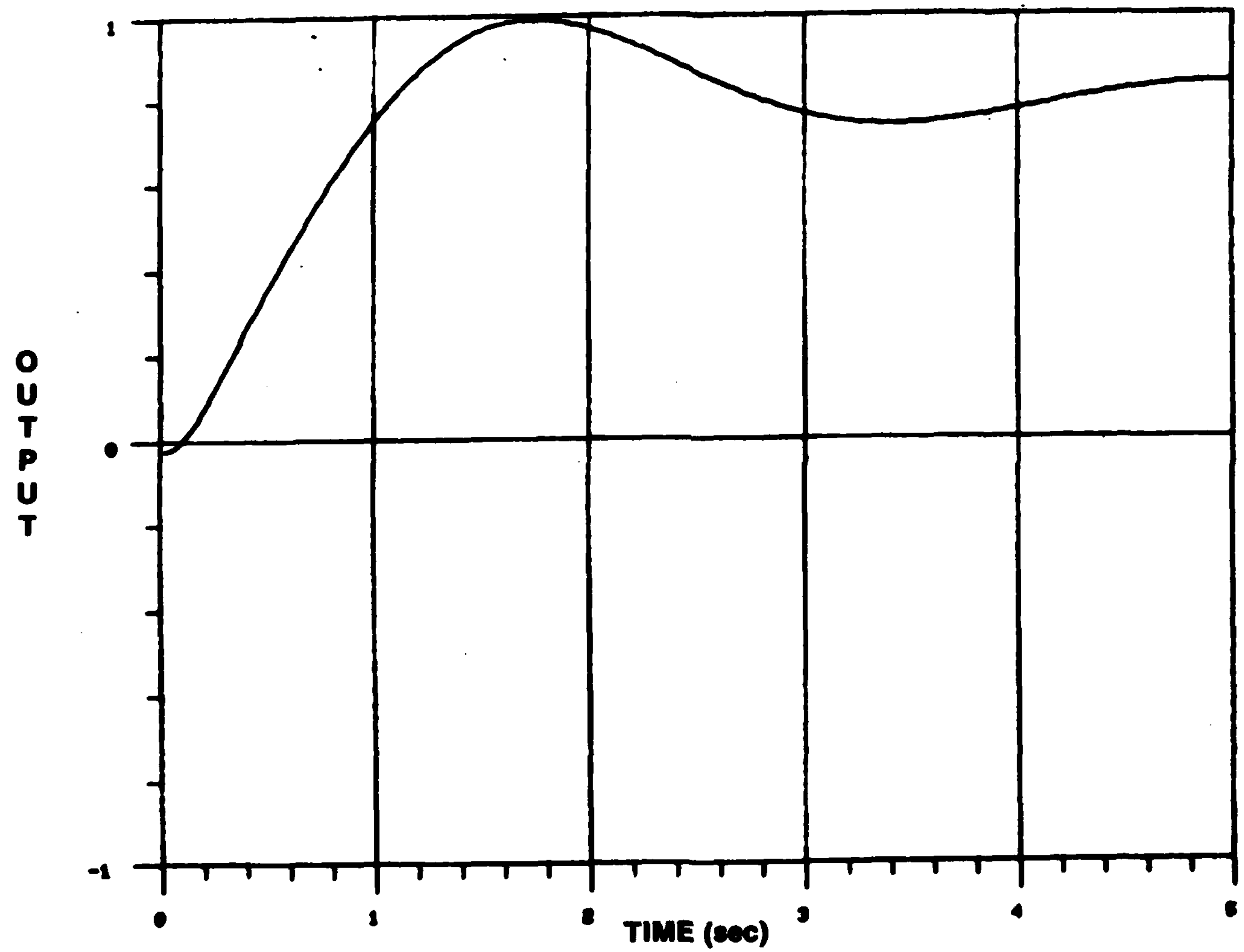


Figure 39. System output response (y), W_1 = 1, -20% variation on all parameters.

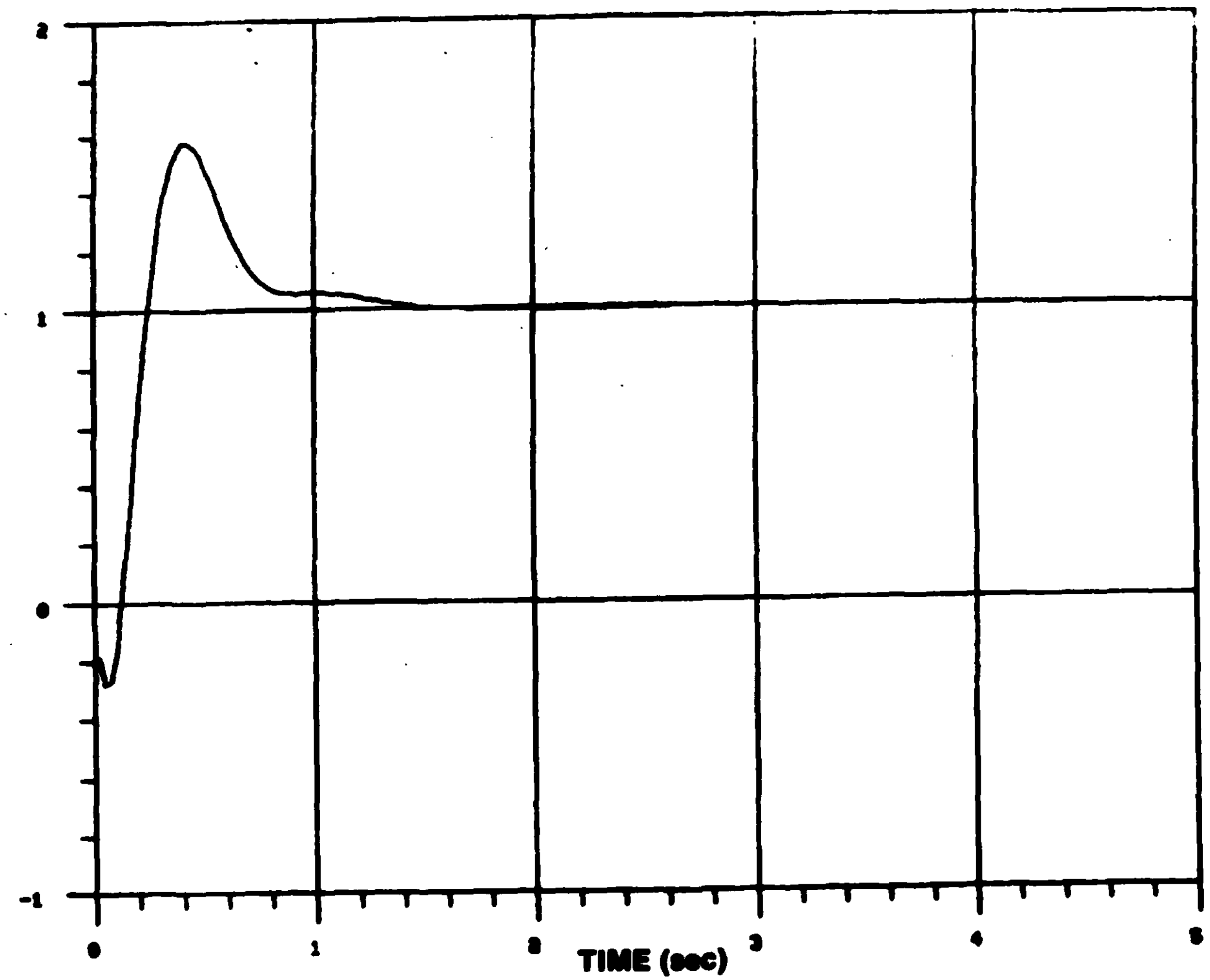


Figure 40. DAC disturbance estimate($\hat{z}_1$), $W_1 = 1$, -20% variation on all parameters.

From these results, the first two questions previously posed can indeed be answered in the affirmative. The answer to the third question would seem to be that the DAC works well with at least up to 20% variation of plant parameters and possibly for larger variations. For a given system, though, this should be thoroughly verified by checking at all critical times along a trajectory, i.e., burnout, apogee, etc.

In order to answer the fourth question, three of the time points shown in *Table 1* were used in the simulation. The roots of Equation (18), which were used to settle out the state

TABLE 3. ROOTS FOR DETERMINING DAC GAIN MATRICES

TIME POINT (SEC) \ ROOT	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
9.85	-5.	-6.	-10.	-10.	-12.	-15.
66.7	-0.5	-0.5	- 1.	- 1.	- 1.5	- 1.5
111.4	-3.	-4.	-7+j2	-7-j2	- 8.	-10.

reconstructor in each case and to calculate the components of the DAC gain matrices, are shown in *Table 3*. The components of $\underline{K}_1$ and $\underline{K}_2$ are shown in *Table 4*.

TABLE 4. DAC GAIN MATRIX COMPONENTS

GAIN VALUE \ TIME POINT (SEC)	9.85	66.7	111.4
k_{11}	-45.77	-1.716	-30.38
k_{21}	-1237.14	-5.85	-660.19
k_{31}	-15951.2	-7.36	-4197.9
k_{41}	-51488.6	-8.16	-13735.5
k_{12}	-155.26	-1.935	-588.51
k_{22}	-300.51	-0.271	-640.07

Three simulation runs were made at each time point:

- with nominal airframe parameters, no disturbance,

- with nominal airframe parameters and a disturbance and

- with a 20% variation on airframe parameters in the direction of increasing flight time, with a disturbance. The results are presented in *Figures 41* through *58*.

From these results and the DAC parameters shown in *Tables 3* and *4*, it is evident that a DAC designed at one point of a trajectory will not perform as well as needed over large portions of the trajectory. Gain switching, similar to an autopilot gain switch program, will be required for DAC implementation.

C. CONCLUSIONS

For this case, with the disturbance at the input, it was possible to find a control $\underline{u}_c$ which could be implemented and which, theoretically, would totally cancel the disturbance. In a practical application, it was found that the control did cancel the disturbance very well, that the DAC would continue to function well within a band about the design point and that, with gain switching, the DAC should perform its function as the plant parameters vary over an entire trajectory. As can be seen from *Table 4* the DAC gains do have a wide range.

For the apogee case, since the system is so sluggish, the DAC did not offer much in the way of disturbance cancellation, i.e., the estimation errors did not settle out rapidly enough. One reason is due to the nearness of the eigenvalues of the $\underline{\tilde{A}}$ matrix to zero. This allowed the large overshoot. However, these eigenvalues had to be maintained in this region because moving them to more negative positions caused an instability to develop. Overall, it might be advantageous to zero out the DAC gains near apogee.

6. RATE LOOP WITH DISTURBANCE ON OUTPUT

A. DAC MODEL DEVELOPMENT

The missile from which this autopilot channel was taken uses an attitude control during boost, so it was of interest to consider the rate loop alone, with a disturbance included, to see if a DAC would be useful in taking out effects due to external rate perturbations.

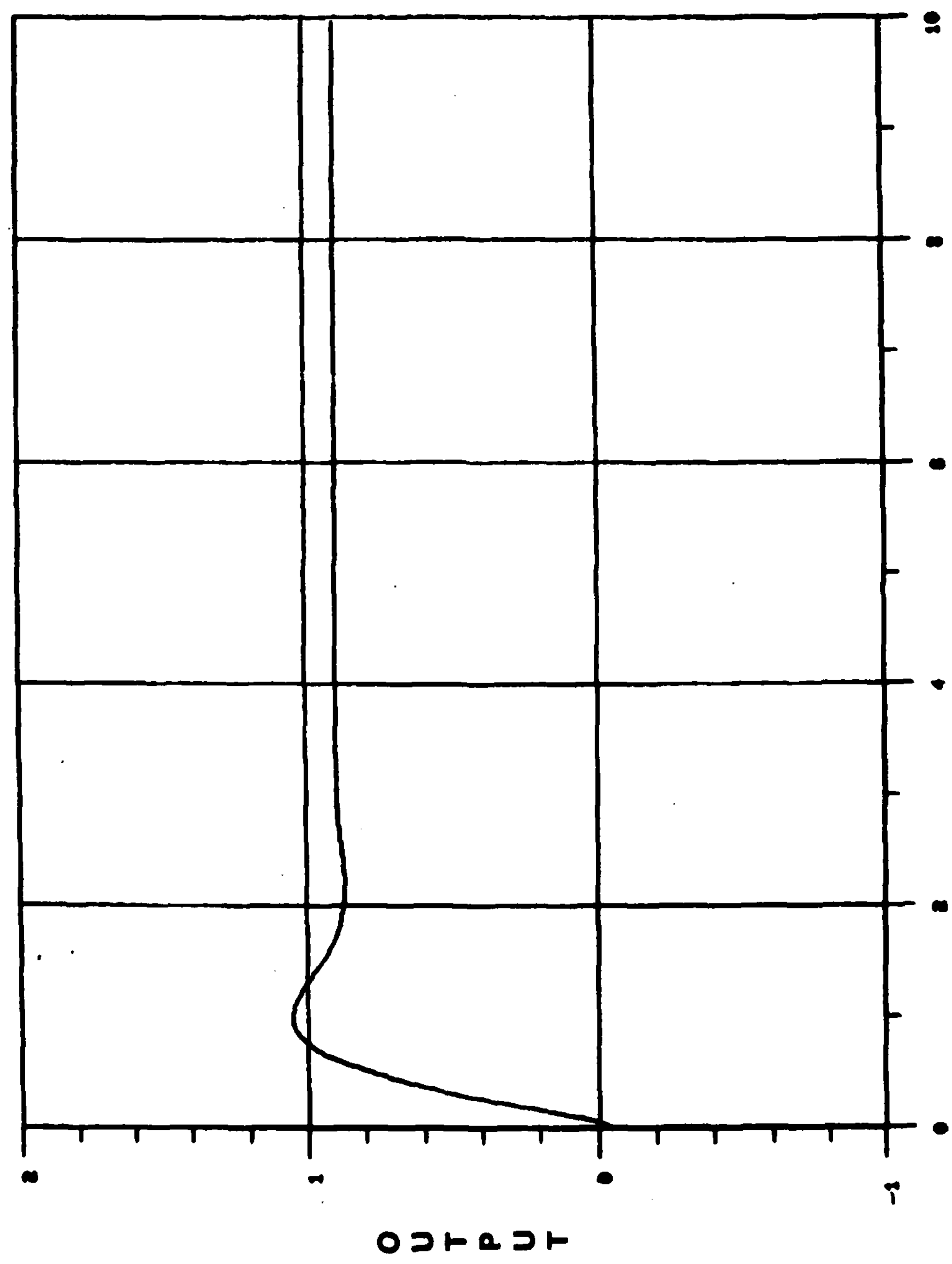


Figure 41. System output response, t_f = 9.85 sec, PGO = 1, W_1 = 0.

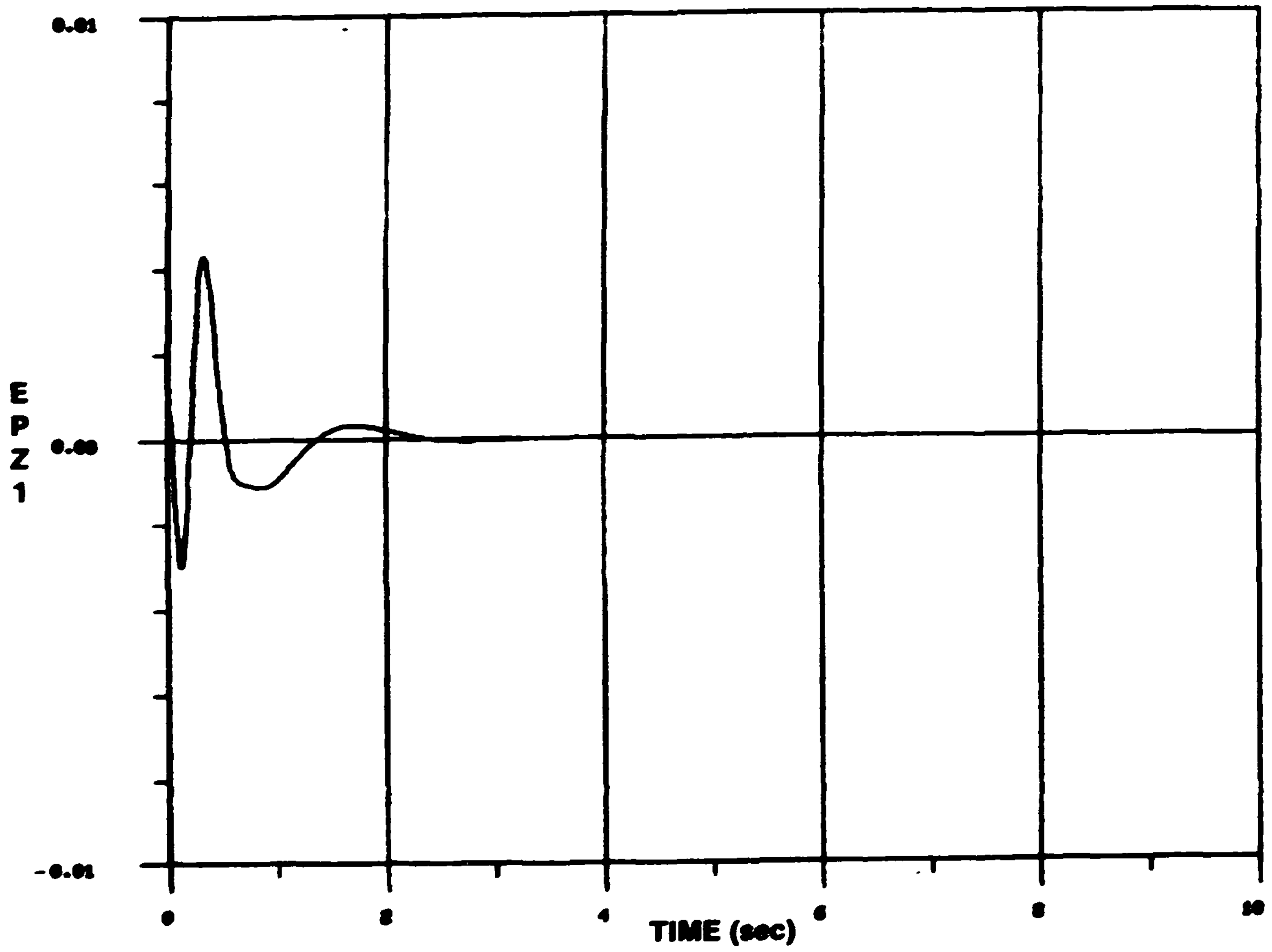


Figure 42. DAC disturbance estimation error, t_f = 9.85 sec, PGO = 1, W_1 = 0.

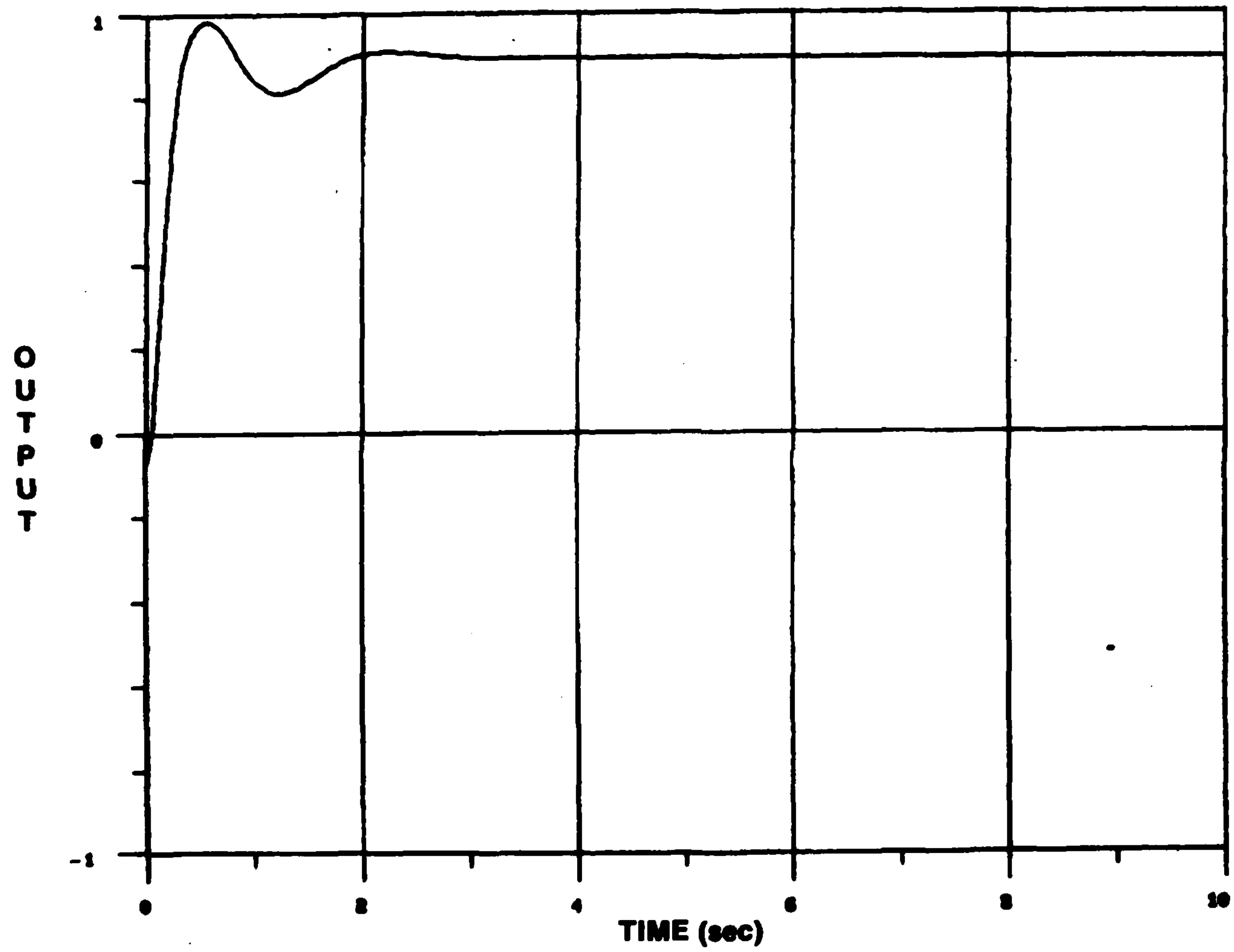


Figure 43. System output response, t_f = 9.85 sec, PGO = 1, W_1 = 1.0.

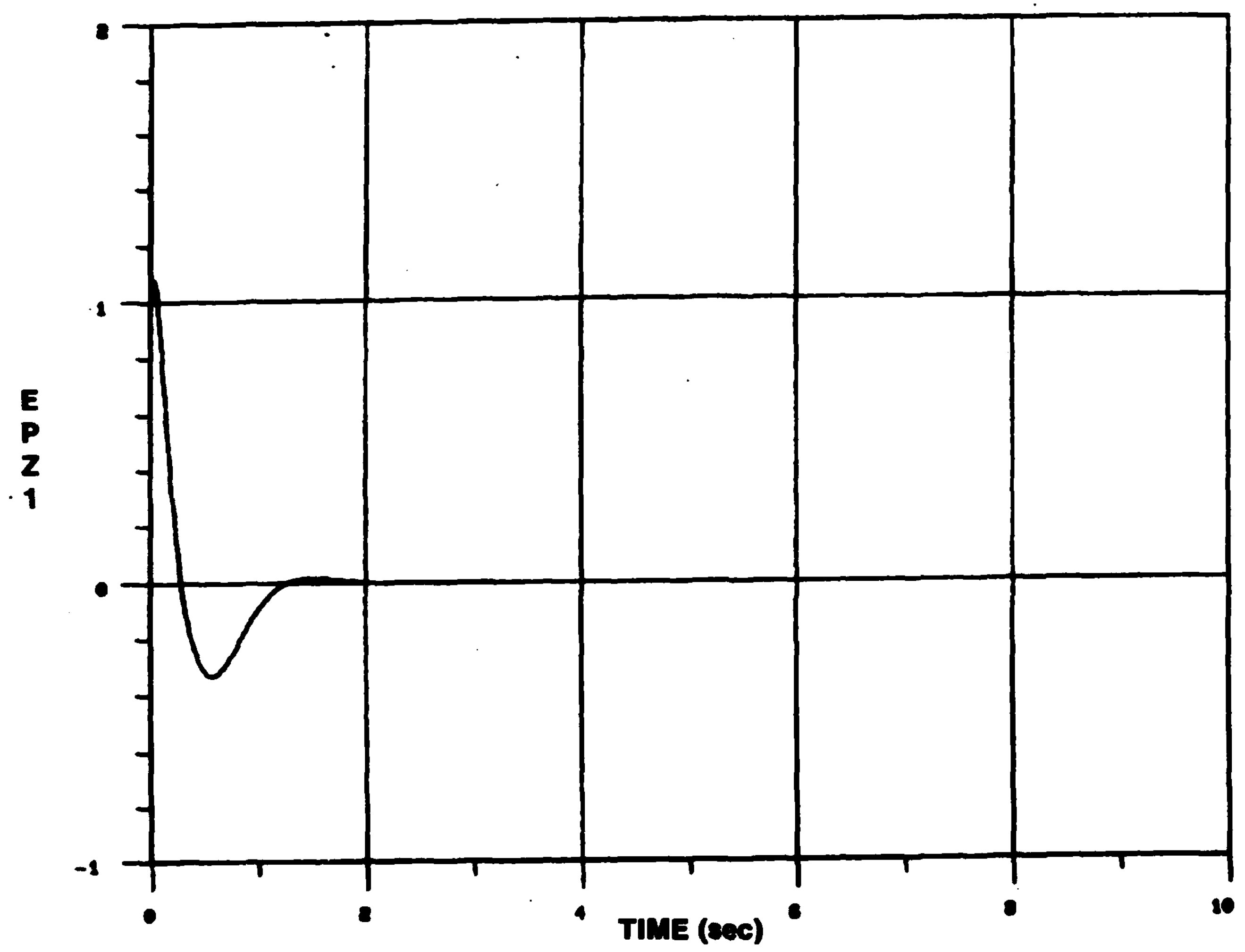


Figure 44. DAC disturbance estimation error, t_f = 9.85 sec, PGO = 1, W_1 = 1.0.

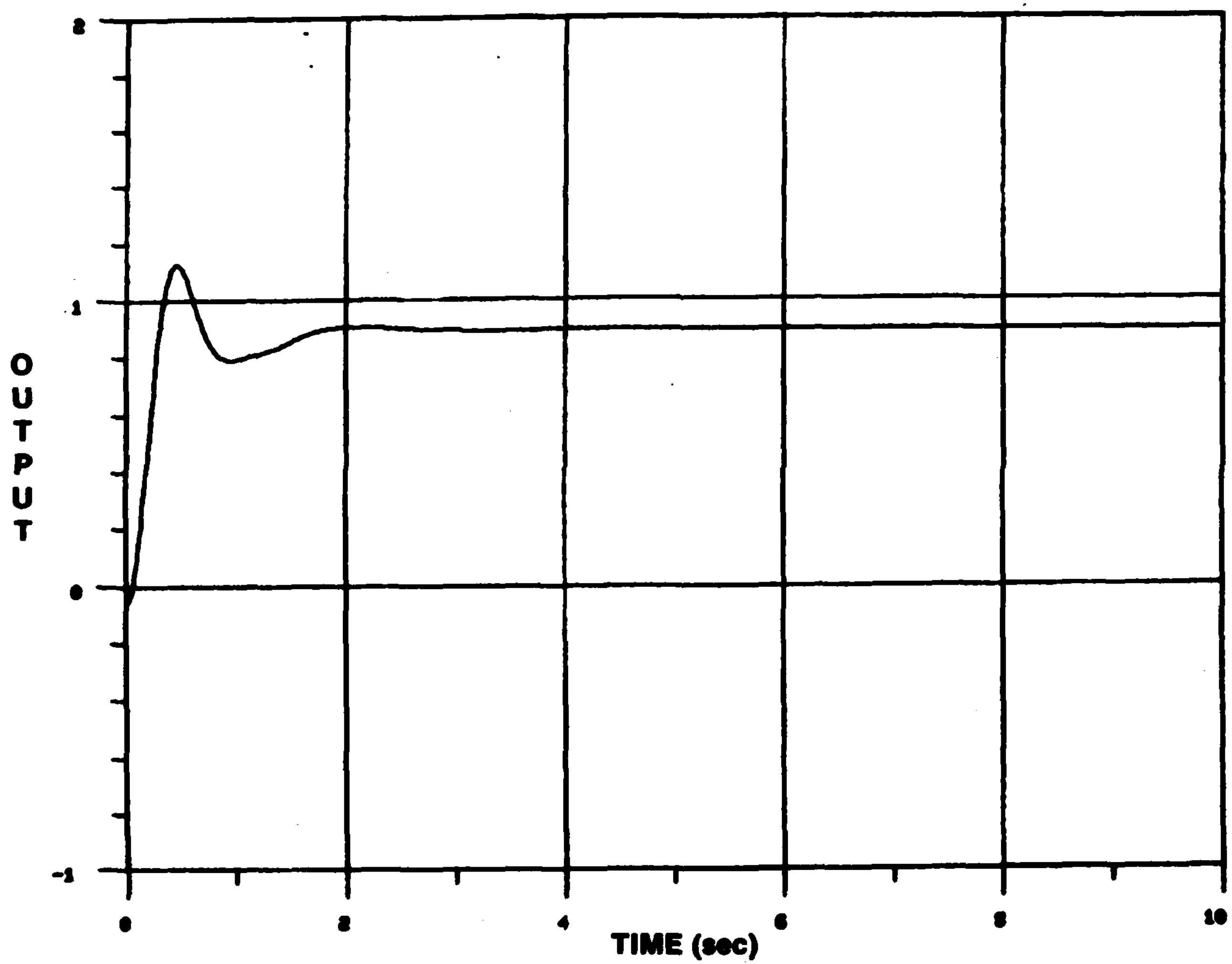


Figure 45. System output response, t_f = 9.85 sec, W_1 = 1.0, -20% variation on airframe parameters.

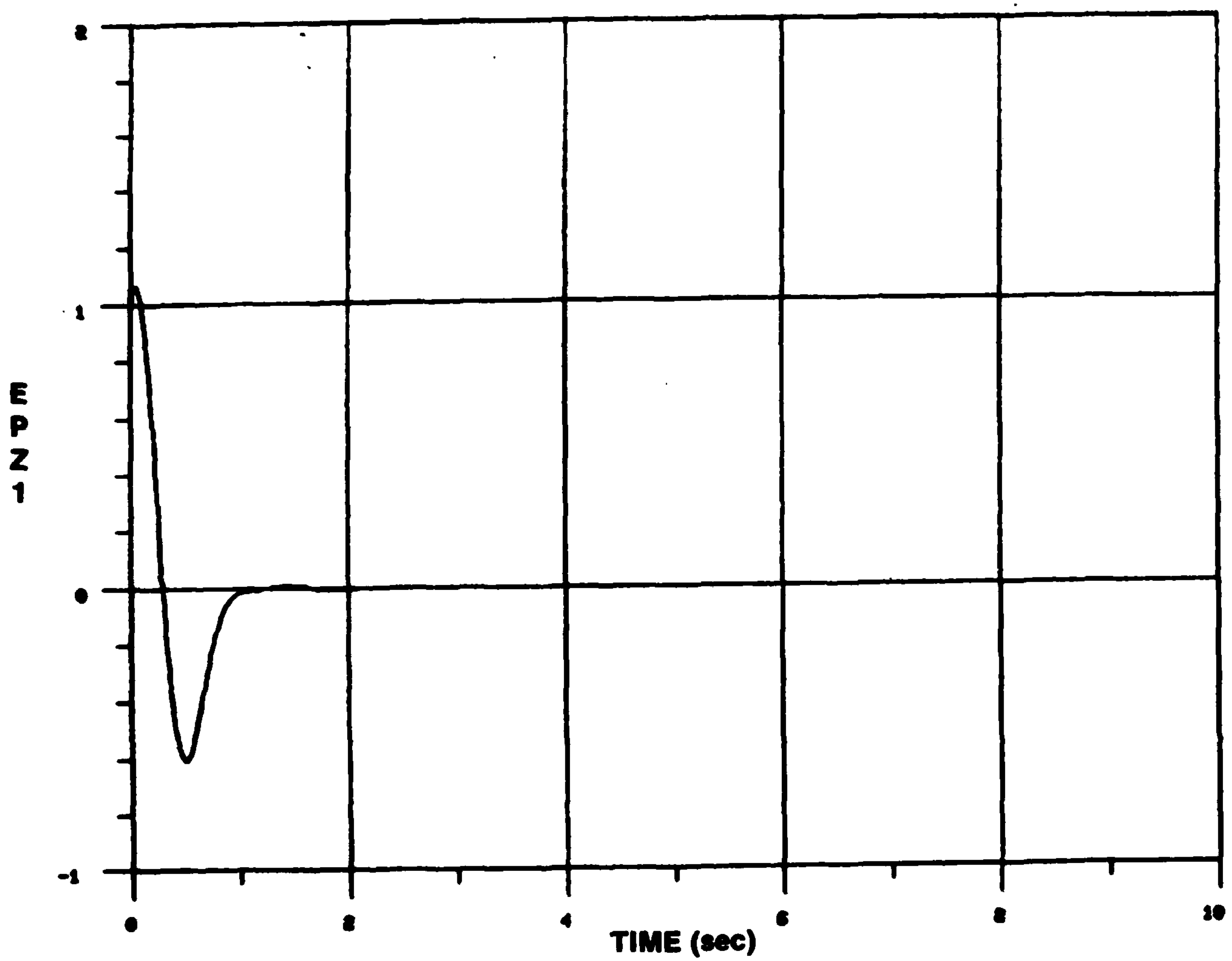


Figure 46. DAC disturbance estimation error, t_f = 9.85 sec, W_1 = 1.0, -20% variation on airframe parameters.

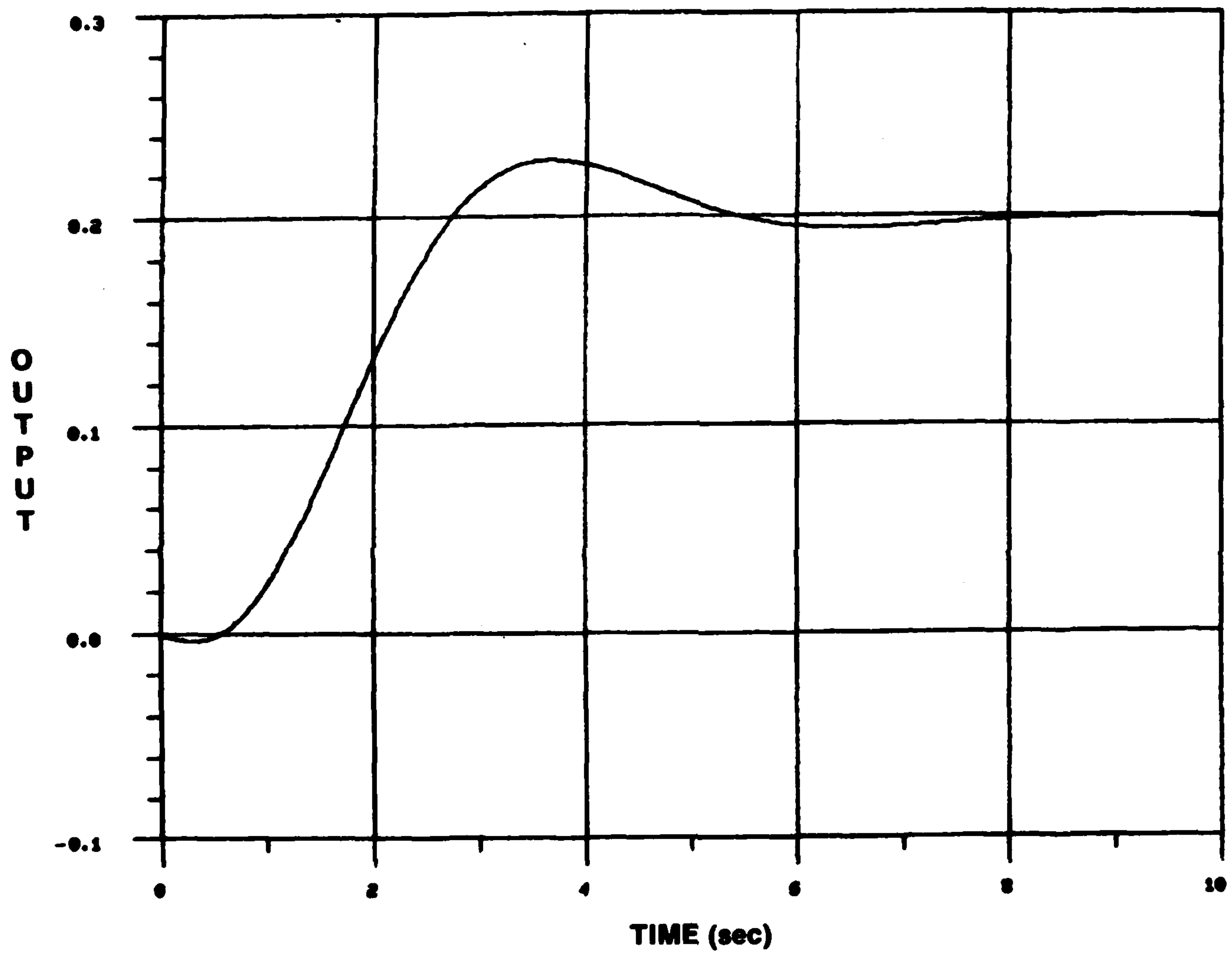


Figure 47. System output response, t_f = 66.7 sec, PGO = 0.5, W_1 = 0.

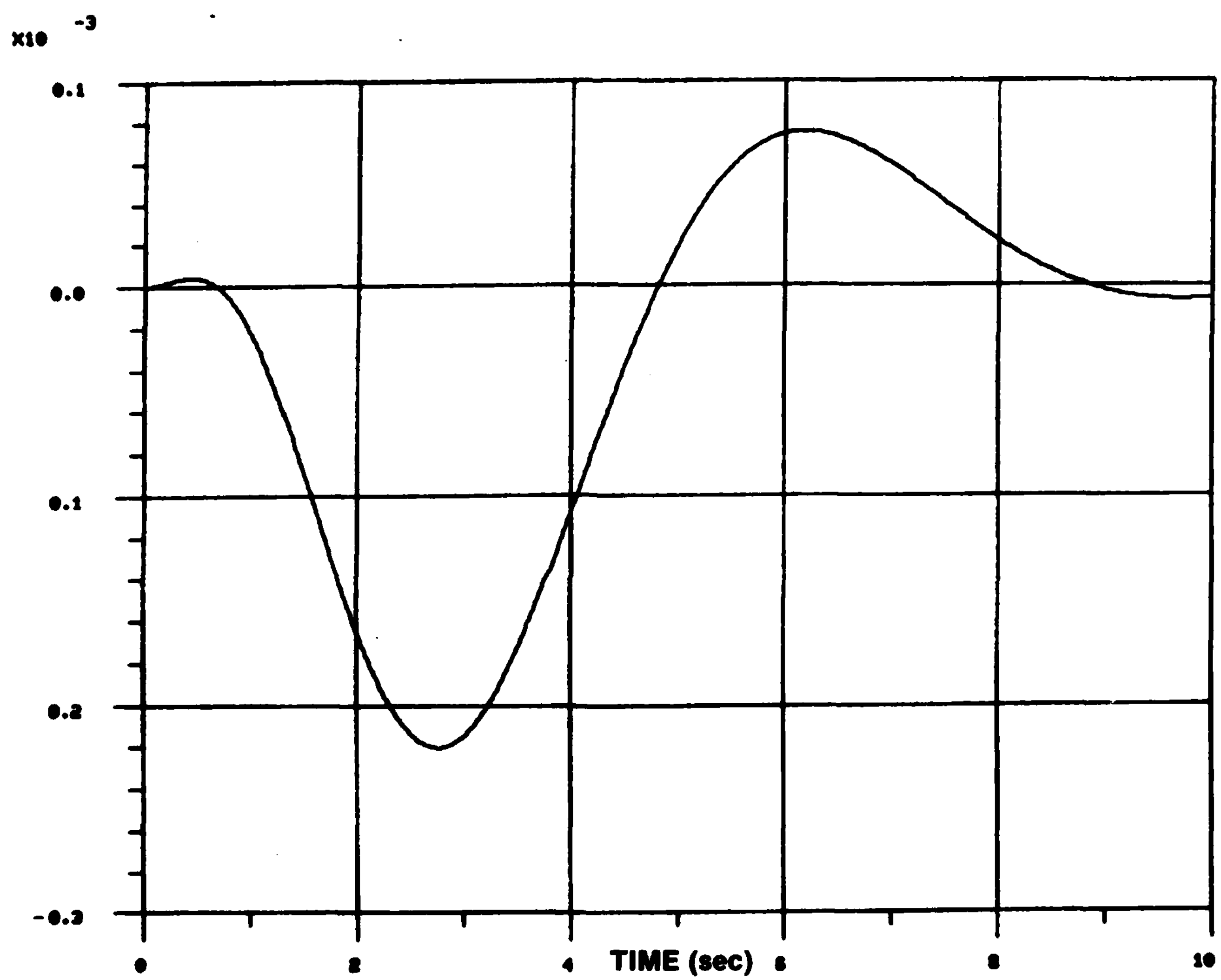


Figure 48. DAC disturbance estimation error, t_f = 66.7 sec, PGO = 0.5, W_1 = 0.5.

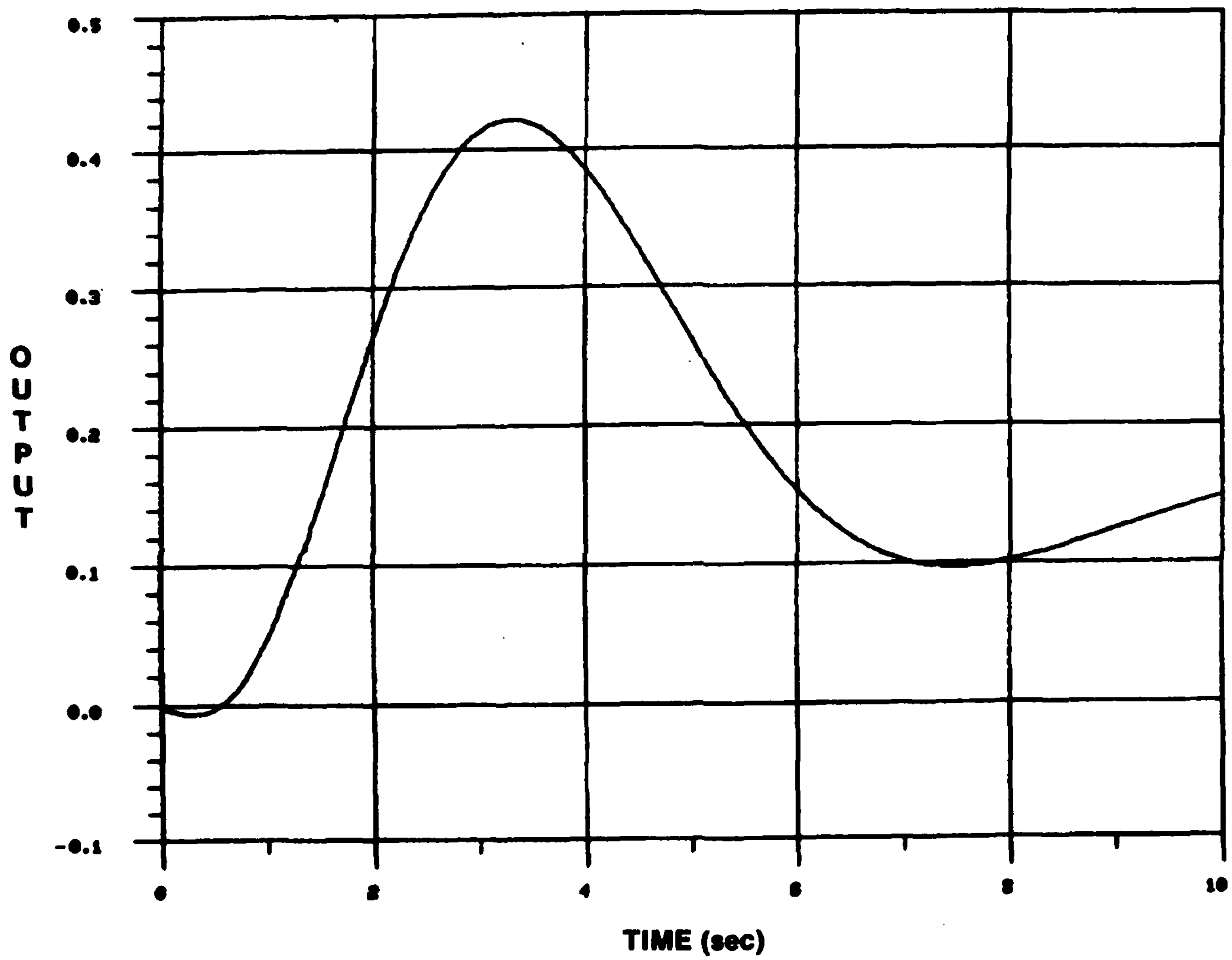


Figure 49. System output response, t_f = 66.7 sec, PGO = 0.5, W_1 = 0.5.

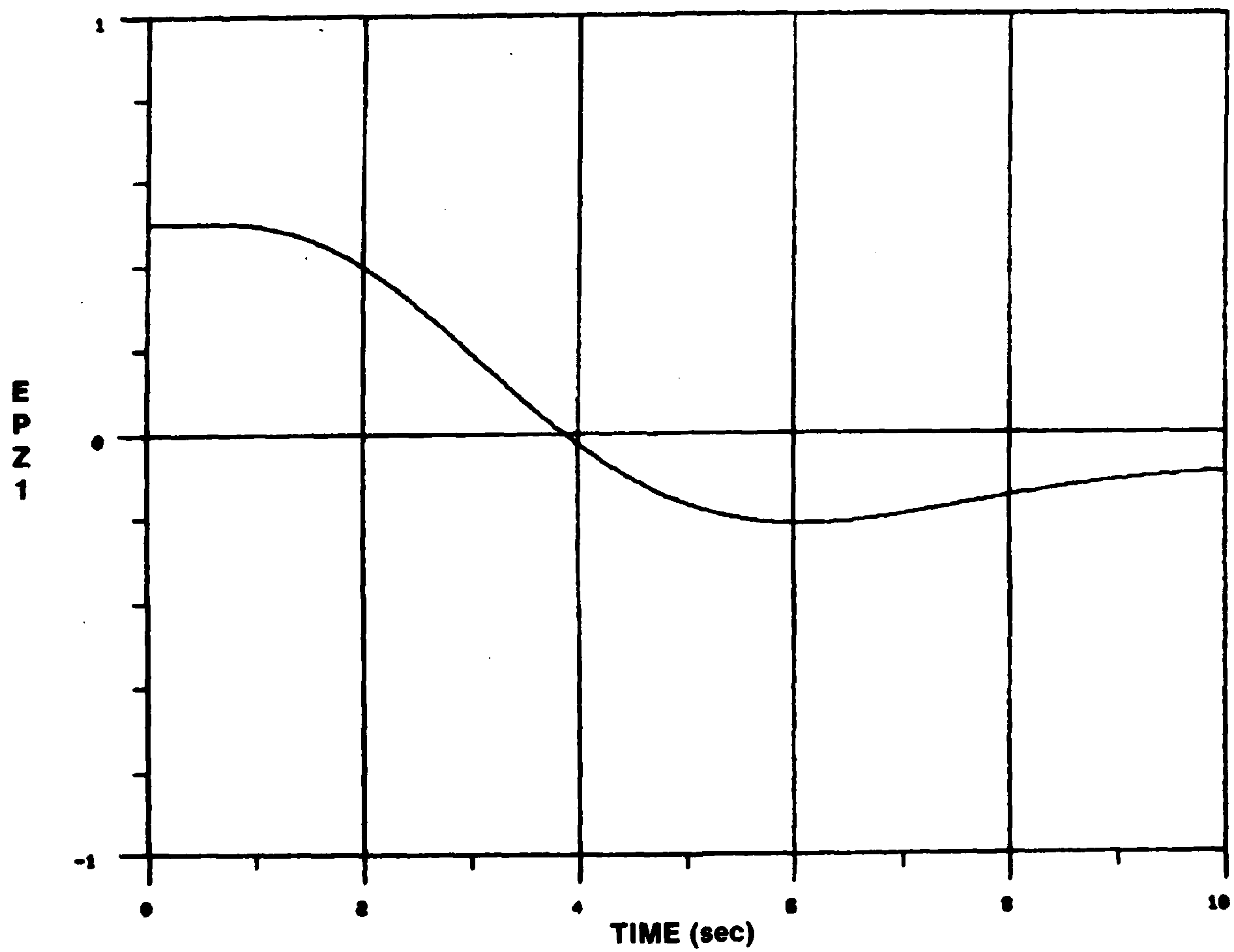


Figure 50. DAC disturbance estimation error, t_f = 66.7 sec, PGO = 0.5, W_1 = 0.5.

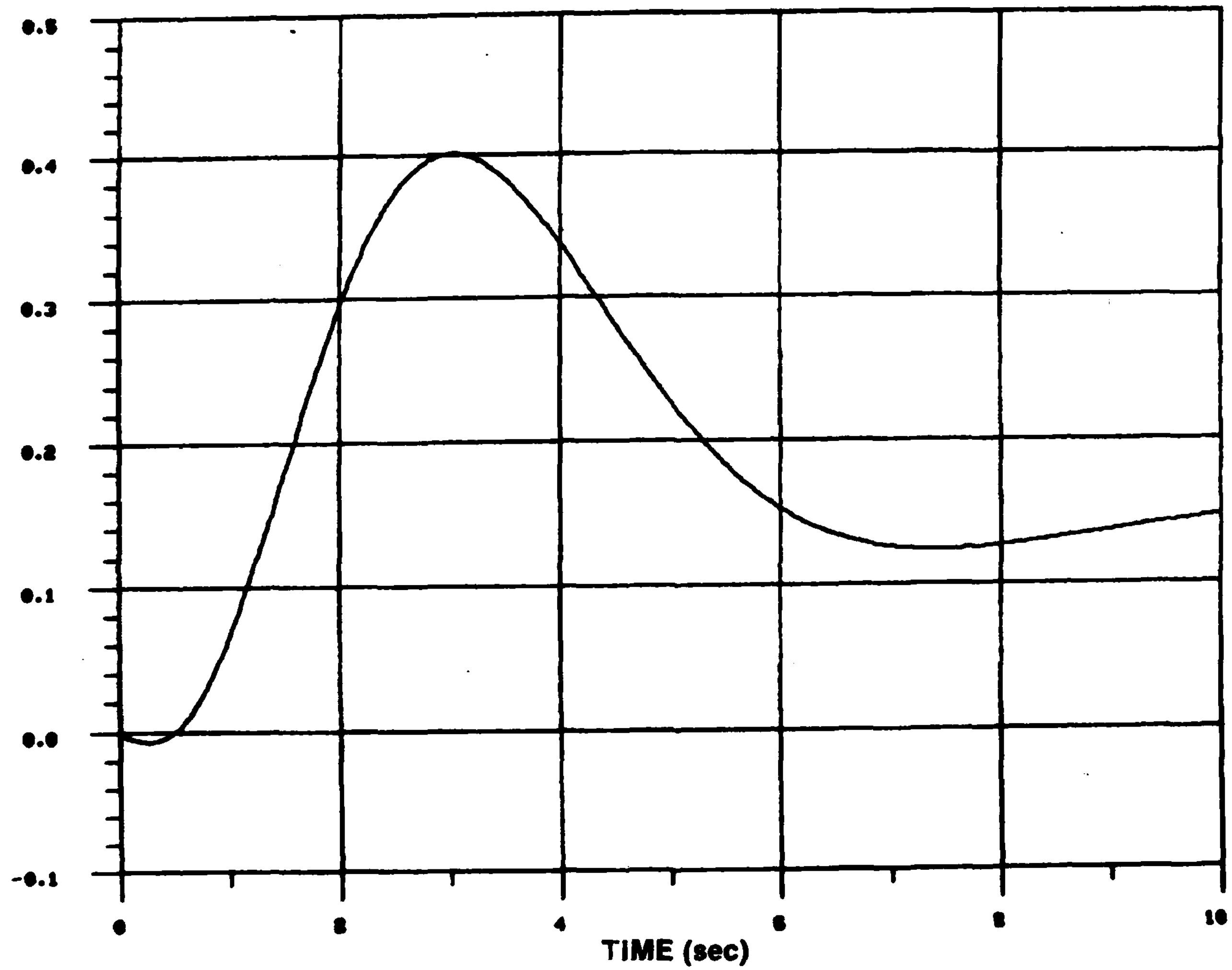


Figure 51. System output response, t_f = 66.7 sec, PGO = 0.5, W_1 = 0.5, + 20% variation on airframe parameters.

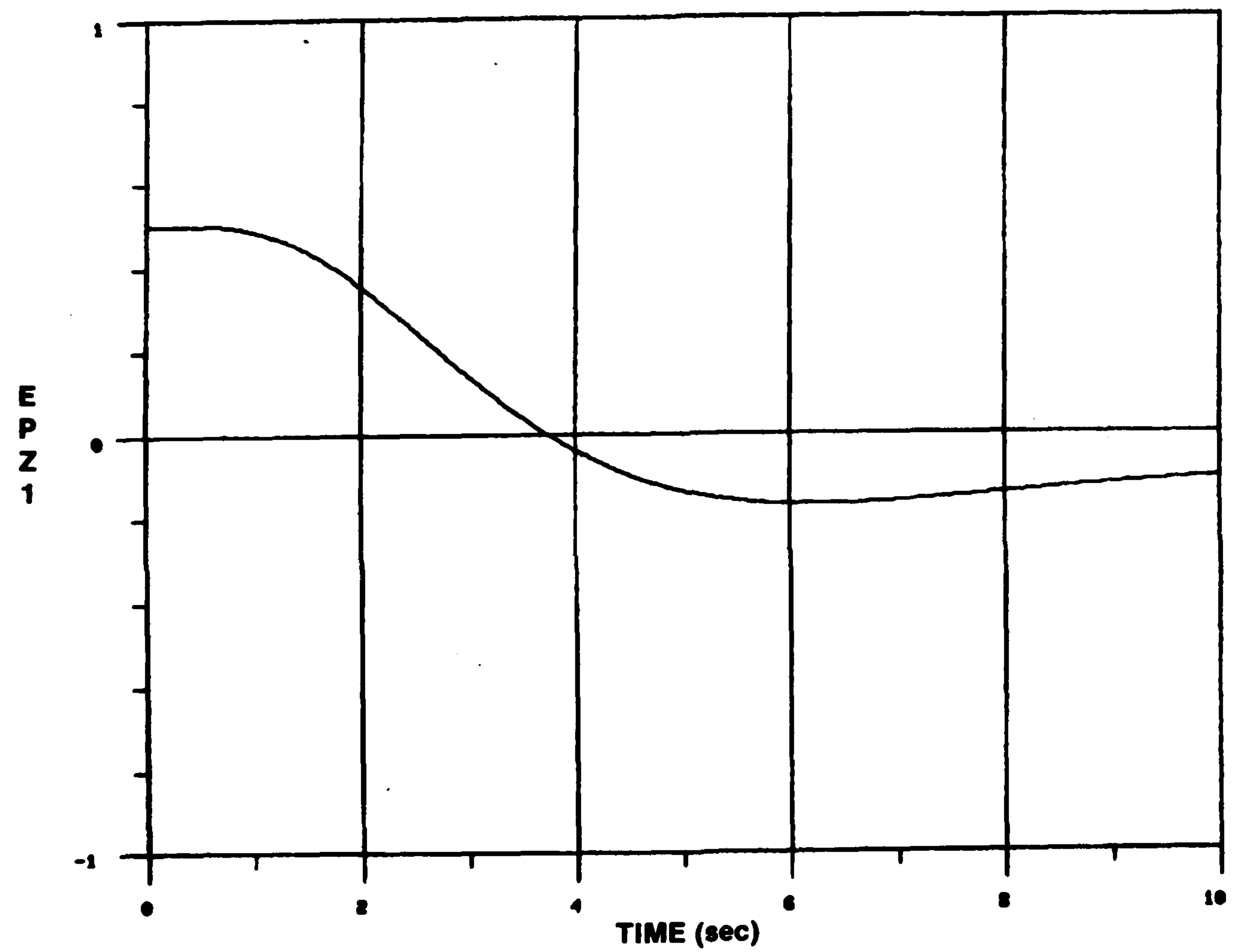


Figure 52. DAC disturbance estimation error, t_f = 66.7 sec, PGO = 0.5, W_1 = 0.5, ± 20% variation on airframe parameters.

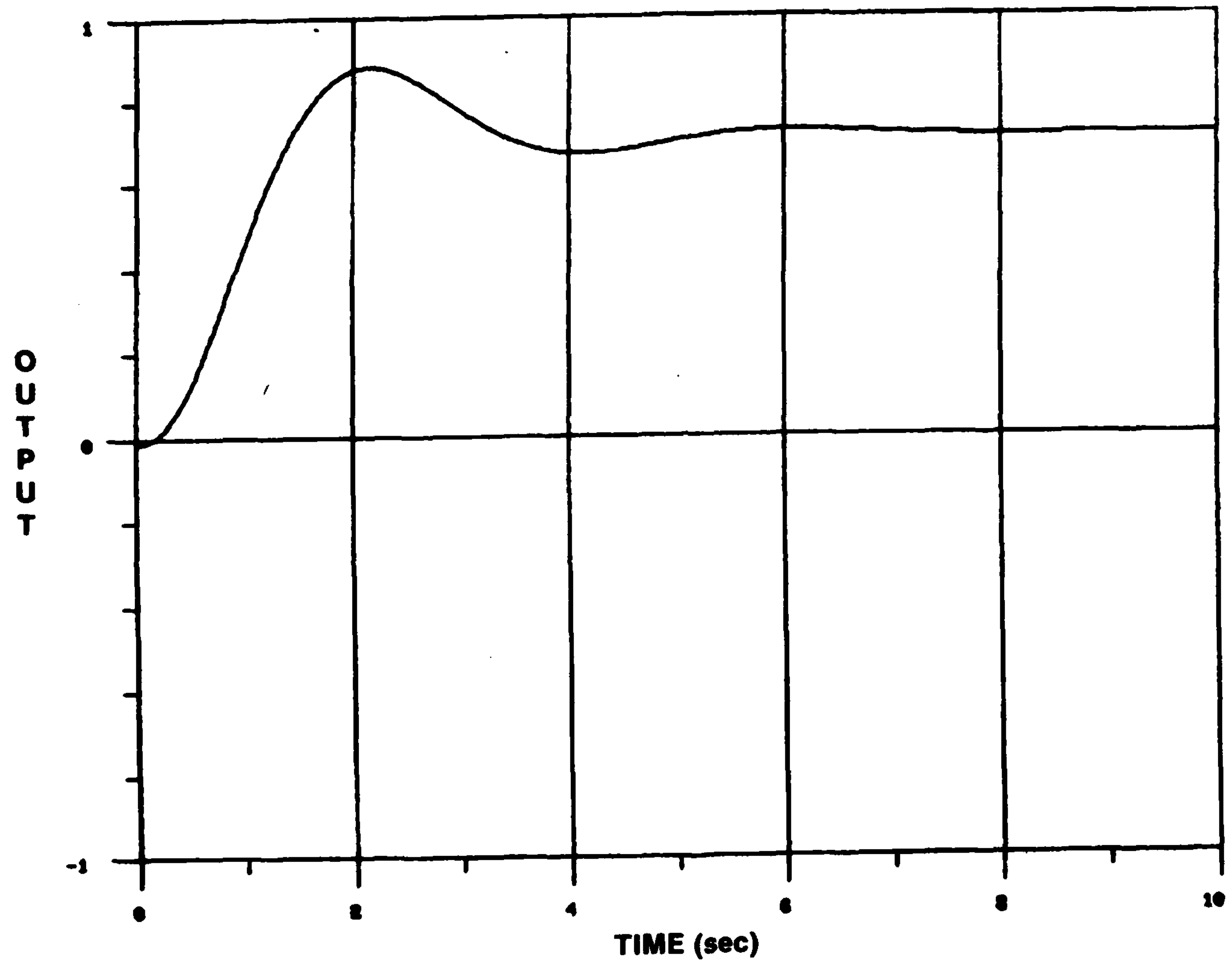


Figure 53. System output response, $t_f = 111.4$ sec, PGO = 1.0, $W_1 = 0.0$.

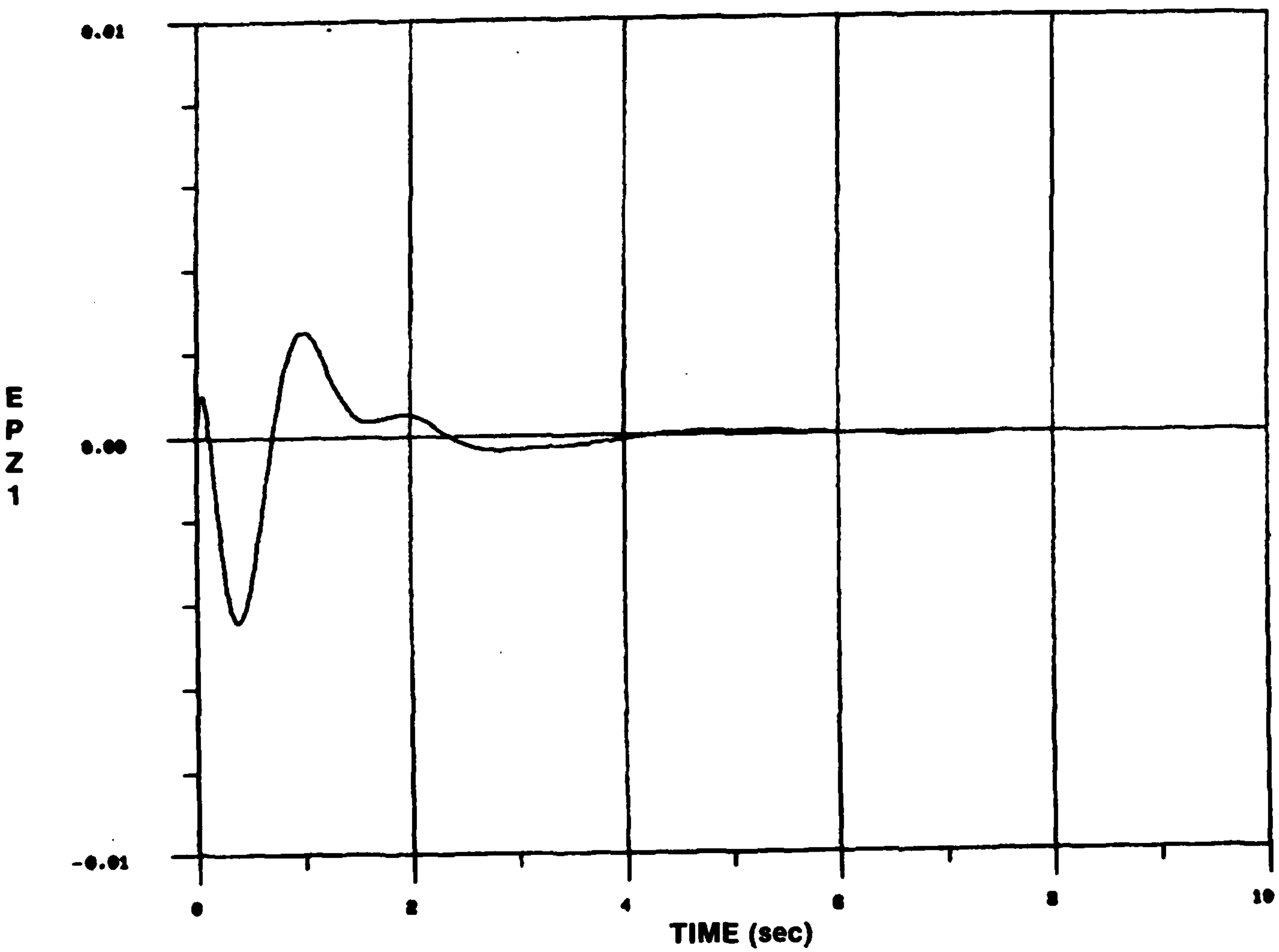


Figure 54. DAC disturbance estimation error, $t_f = 111.4$ sec, PGO = 1.0, $W_1 = 0.0$.

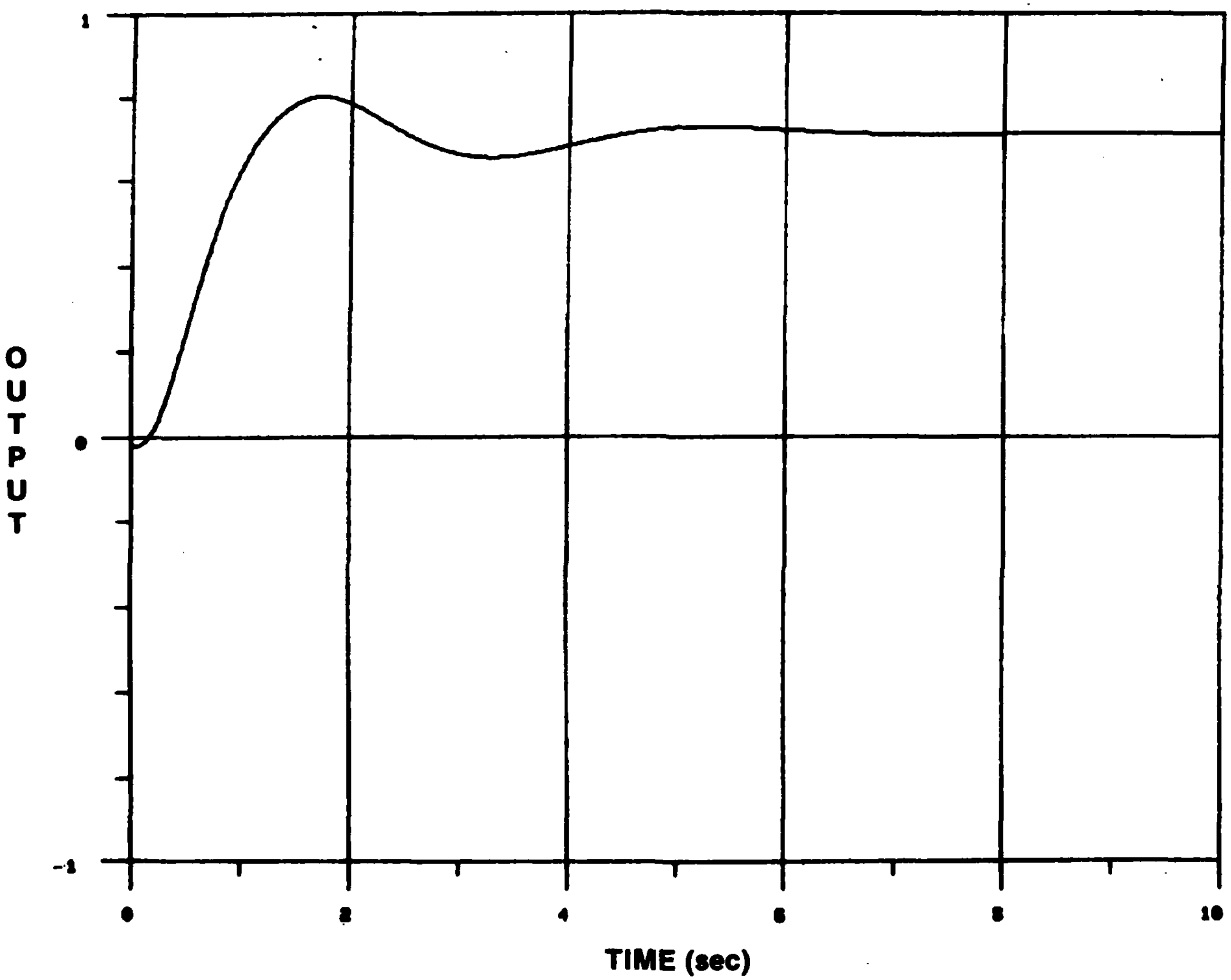


Figure 55. System output response, t_f = 111.4 sec, PGO = 1.0, W_1 = 1.0.

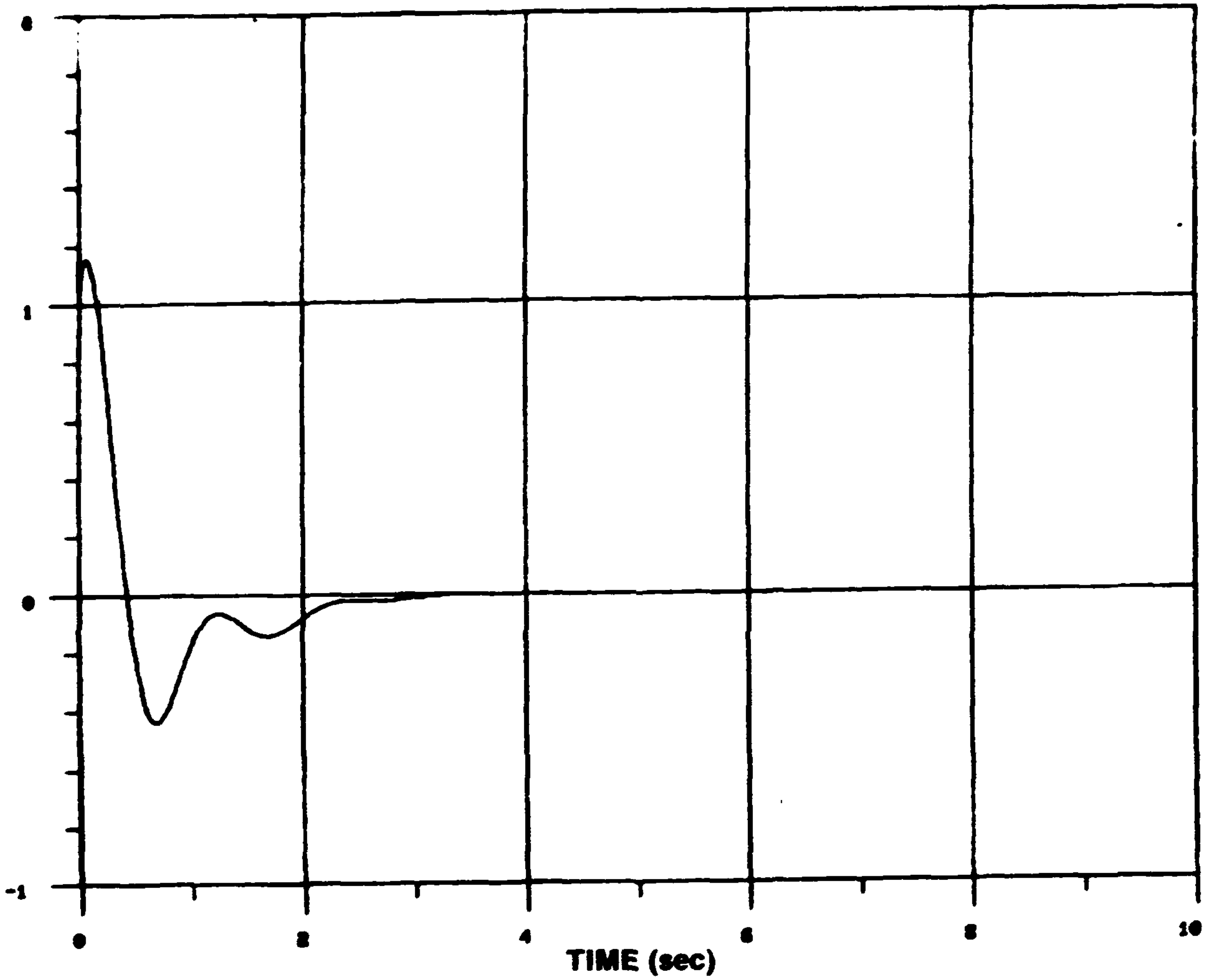


Figure 56. DAC disturbance estimation error, t_f = 111.4 sec, PGC = 1.0, W_1 = 1.0.

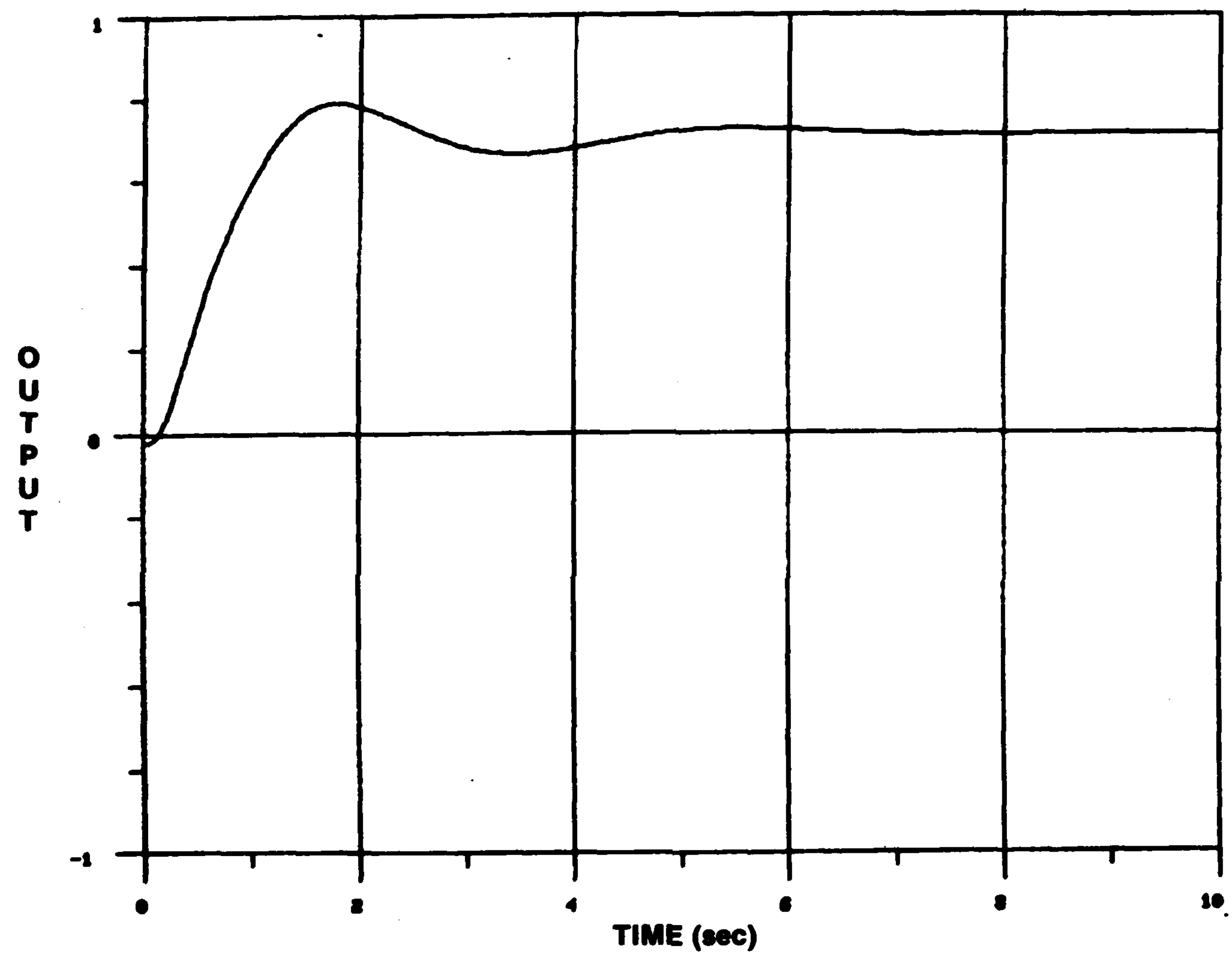


Figure 57. System output response, t_f = 111.4 sec, PGO = 1.0, W_1 = 1.0, +20% variation on airframe parameters.

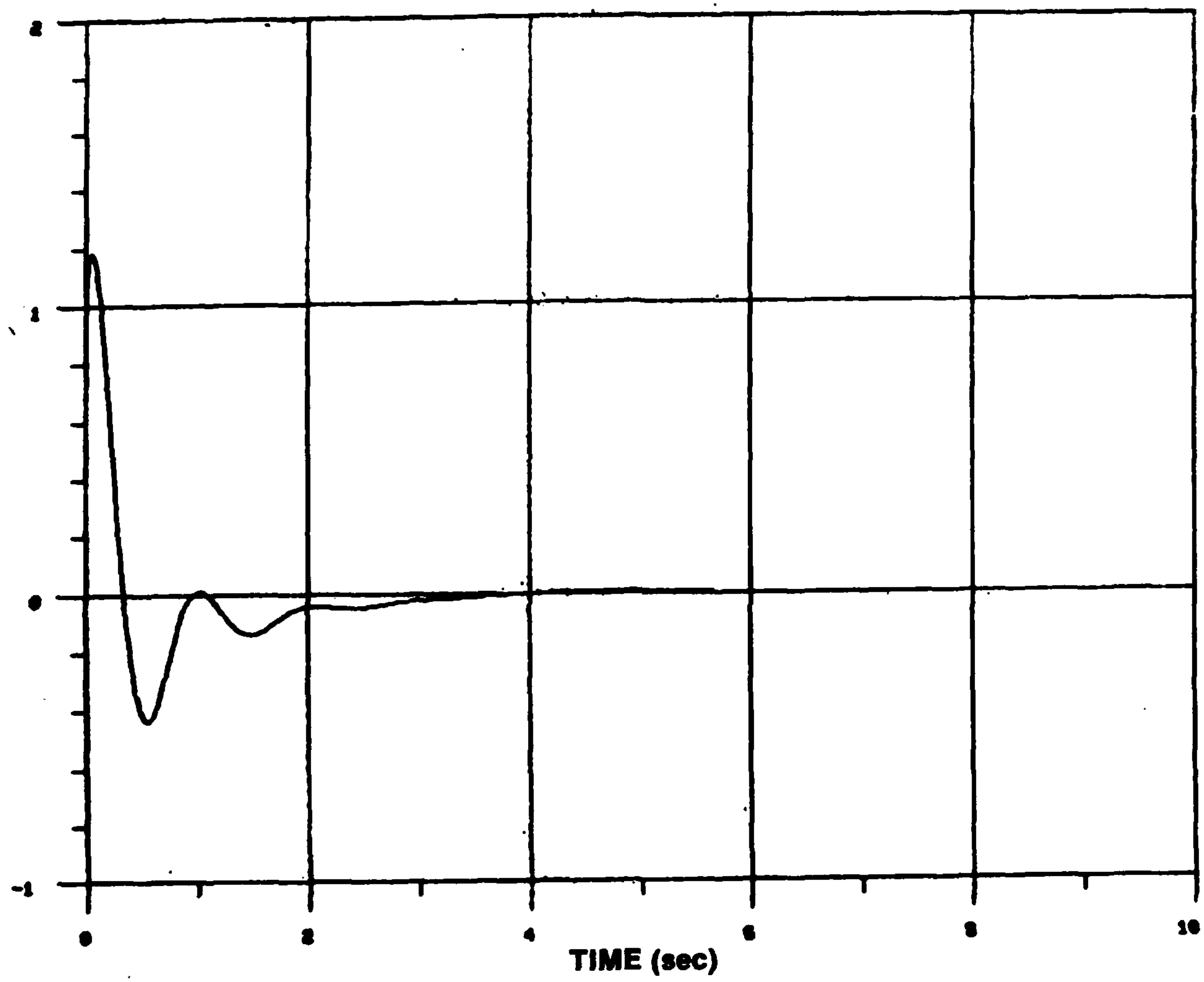


Figure 58. DAC disturbance estimation error, t_f = 111.4 sec, PGO = 1.0, W_1 = 1.0, +20% variation on airframe parameters.

The rate loop shown in *Figure 2* was rearranged and simplified as shown in *Figure 59*. The compensation term was reduced to C_R since no actuator dynamics are considered. The disturbance is shown as a rate imposed on the airframe in addition to that due to a given fin deflection. Therefore, the total body rate,

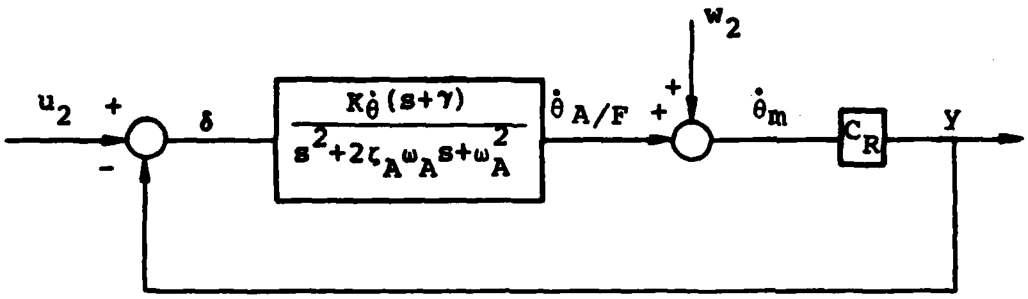


Figure 59. Rate loop block diagram.

assumed to be measured by an ideal rate gyro, would be

$$\dot\theta_m = \dot\theta_{A/F} + w_2 \quad .$$

From *Figure 59* one has

$$\frac{\dot\theta_{A/F}(s)}{\delta(s)} = \frac{K_{\dot\theta}(s+\gamma)}{s^2+2\zeta_A\omega_A s+\omega_A^2} \quad .$$

Therefore,

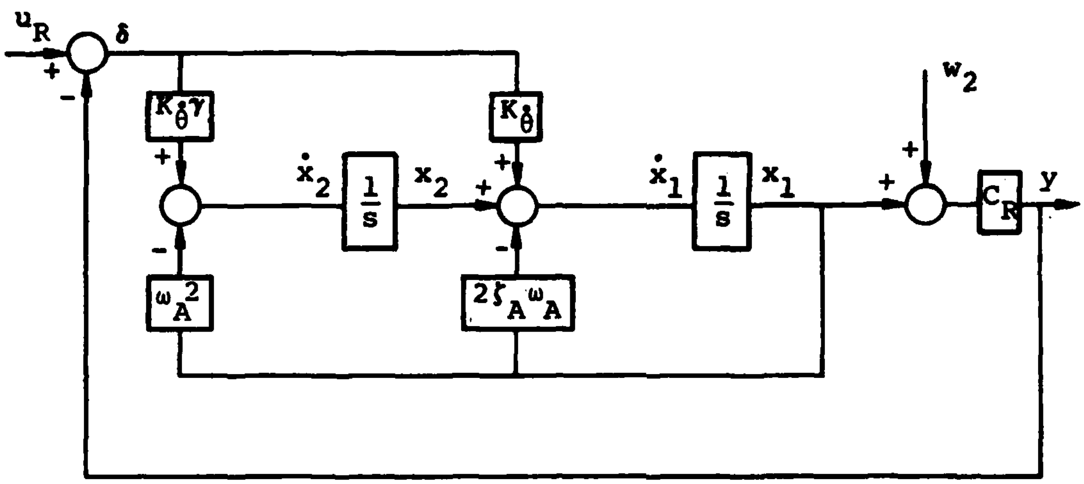
$$s^2\dot\theta_{A/F}(s) + 2\zeta_A\omega_A s\dot\theta_{A/F}(s) + \omega_A^2\,\dot\theta_{A/F}(s) =$$

$$K_{\dot\theta}s\delta(s) + K_{\dot\theta}\gamma\delta(s)$$

and

$$\dot\theta_{A/F}(s) = \frac{1}{s}\,[K_{\dot\theta}\delta(s) - 2\zeta_A\omega_A\dot\theta_{A/F}(s) + \frac{1}{s}\,(K_{\dot\theta}\gamma\delta(s) - \omega_A^2\dot\theta_{A/F}(s))] \, .$$

This appears, along with the rest of the loop, as



Writing the state space equations directly from this and substituting for δ gives:

$$\dot x_2 = K_{\dot\theta}\gamma\delta - \omega_A^2 x_1 = -(K_{\dot\theta}\gamma C_R - \omega_A^2)\, x_1 + K_{\dot\theta}\gamma U_R - K_{\dot\theta}\gamma C_R w_2$$

$$\dot x_1 = x_2 + K_{\dot\theta}\delta - 2\zeta_A\omega_A x_1 = -(K_{\dot\theta}C_R + 2\zeta_A\omega_A)\, x_1 + x_2 + K_{\dot\theta}U_R - K_{\dot\theta}C_R w_2$$

$$y = (x_1 + w_2)\, C_R$$

$$\delta = u_R - y \quad . \tag{21}$$

Expressing these in the form (1),

$$\begin{pmatrix} \dot x_1 \\ \dot x_2 \end{pmatrix} = \begin{bmatrix} -(K_{\dot\theta}C_R + 2\zeta_A\omega_A) & 1 \\ -(K_{\dot\theta}\gamma C_R + \omega_A^2) & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} K_{\dot\theta} \\ K_{\dot\theta}\gamma \end{bmatrix} \underline{u}_R + \begin{bmatrix} -K_{\dot\theta}C_R \\ -K_{\dot\theta}\gamma C_R \end{bmatrix} \underline{w}_2$$

$$\underline{y} = [C_R \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]\,\underline{u}_R + [C_R]\,\underline{w}_2 \ .$$

Now, in this case, $\underline{F} \equiv \underline{B}\,\underline{\Gamma}$ for $\underline{\Gamma} = [-C_R]$. Therefore, the theoretical total absorption controller would be $\underline{u}_c = [C_R]\,\underline{w}_2$. If this is implemented, however, it does not remove $\underline{w}_2$ from the output, $\underline{y}$. But, the desired action in this case is to remove any disturbance rates imposed on the missile so that it will maintain a given attitude during boost. On examining the plant, it can be seen that the disturbance rate $\underline{w}_2$ imposed on the body can be related to an "equivalent" fin deflection δ_D, i.e., it can be considered as an additional body rate which would have resulted from an additional fin deflection command δ_D. In other words,

$$\dot{\theta}_m = \dot{\theta}_{A/F} + w_2 = (\delta + \delta_D)\,\frac{\dot{\theta}}{\delta} \quad .$$

So, if the proper gain can be found, and if a state observer can be designed which will reconstruct $\hat{z}_1$, then $u_C = -C_{RG}\hat{z}_1$ can perhaps be used as a partial absorption (minimization) control on the effects of $\underline{w}_2$, where C_{RG} is the sought gain.

Again, Equation (4) is used to obtain the gain matrices $\underline{K}_1$, $\underline{K}_2$. In this case,

$$\underline{\dot{\varepsilon}} = \left[\begin{array}{c|c} \begin{bmatrix} -(K_{\dot{\theta}}C_R + 2\zeta_A\omega_A) & 1 \\ -(K_{\dot{\theta}}\gamma C_R + \omega_A^2) & 0 \end{bmatrix} + \begin{bmatrix} k_{11} \\ k_{21} \end{bmatrix}[C_R \;\; 0] & \begin{bmatrix} k_{11} - K_{\dot{\theta}} \\ k_{21} - K_{\dot{\theta}}\gamma \end{bmatrix} [C_R \;\; 0] \\ \hline \begin{bmatrix} k_{12} \\ k_{22} \end{bmatrix} [C_R \;\; 0] & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} k_{12} \\ k_{22} \end{bmatrix} [C_R \, 0] \end{array}\right] \underline{\varepsilon} + \begin{pmatrix} \underline{0} \\ \underline{\sigma} \end{pmatrix} \cdot \tag{22}$$

so,

$$\begin{pmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_z \end{pmatrix} = \begin{bmatrix} k_{11}C_R - 2\zeta_A\omega_A - K_{\dot{\theta}}C_R & 1 & (k_{11} - K_{\dot{\theta}})C_R & 0 \\ k_{21}C_R - \omega_A^2 - K_{\dot{\theta}}\gamma C_R & 0 & (k_{21} - K_{\dot{\theta}}\gamma)C_R & 0 \\ k_{12}C_R & 0 & k_{12}C_R & 1 \\ k_{22}C_R & 0 & k_{22}C_R & 0 \end{bmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_z \end{pmatrix} + \begin{pmatrix} \underline{0} \\ \underline{\sigma} \end{pmatrix} . \tag{23}$$

Let Equation (23) be written

$$\dot{\underline{\varepsilon}} = \underline{\tilde{B}}\underline{\varepsilon} + \begin{pmatrix} \underline{o} \\ \underline{\sigma} \end{pmatrix}$$

and $\underline{\tilde{B}}$ be written as

$$\underline{\tilde{B}} = \begin{bmatrix} B_1 & 1 & B_5 & 0 \\ B_2 & 0 & B_6 & 0 \\ B_3 & 0 & B_7 & 1 \\ B_4 & 0 & B_8 & 0 \end{bmatrix}$$

Solve for the eigenvalues of $\underline{\tilde{B}}$.

$$\det |\underline{\tilde{B}} - \lambda \underline{I}| = 0$$

$$\det |\underline{\tilde{B}} - \lambda \underline{I}| = \begin{vmatrix} B_1-\lambda & 1 & B_5 & 0 \\ B_2 & -\lambda & B_6 & 0 \\ B_3 & 0 & B_7-\lambda & 1 \\ B_4 & 0 & B_8 & -\lambda \end{vmatrix} = 0 .$$

This can be expanded to give

$$\begin{aligned} &\lambda^4 - (B_1+B_3)\lambda^3 + (B_1B_3-B_1B_4-B_3B_5-B_2)\lambda^2 \\ &+ (B_1B_4-B_4B_5+B_2B_3-B_3B_6)\lambda \\ &+ (B_4B_2-B_6B_4) = 0 . \end{aligned} \tag{24}$$

If the desired roots of Equation (24) are λ_1, λ_2, λ_3, λ_4, then the desired characteristic equation is

$$(\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)(\lambda-\lambda_4) = 0 . \tag{25}$$

Expanding Equation (25), equating coefficients of like powers of λ between Equations (24) and (25), substituting back in for B_1 through B_8 and solving for k_{11}, k_{21}, k_{12}, k_{22} gives

$$k_{11} = (2\zeta_A\omega_A - k_{12}C_R + \lambda_1+\lambda_2+\lambda_3+\lambda_4 + K_{\dot{\theta}}C_R)/C_R$$

$$k_{21} = [2\zeta_A\omega_A C_R(k_{12}-k_{22}) + k_{11}k_{12}C_R^2 - \omega_A^2 - K_{\dot{\theta}}k_{22}C_R^2$$

$$+ \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4] \;/\; (-C_R)$$

$$k_{12} = (-2\zeta_A\omega_A C_R k_{22} + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4 + \lambda_1\lambda_2\lambda_3$$

$$+ \lambda_1\lambda_2\lambda_4) \;/\; (C_R\omega_A^2)$$

$$k_{22} = -\lambda_1\lambda_2\lambda_3\lambda_4 \;/\; C_R\omega_A^2$$

Again, the λ's are picked such that $\epsilon(t) \to 0$ rapidly. With these, the gains can be determined.

The full-dimensional observer, in the form of Equation (3), for this case is

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \end{bmatrix} = \begin{bmatrix} k_{11}C_R - 2\zeta_A\omega_A & 1 & k_{11}C_R & 0 \\ k_{21}C_R - \omega_A^2 & 0 & k_{21}C_R & 0 \\ k_{12}C_R & 0 & k_{12}C_R & 1 \\ k_{22}C_R & 0 & k_{22}C_R & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} - \begin{bmatrix} k_{11} \\ k_{21} \\ k_{12} \\ k_{22} \end{bmatrix} \underline{y} + \begin{bmatrix} K_{\dot{\theta}} \\ K_{\dot{\theta}}\gamma \\ 0 \\ 0 \end{bmatrix} \underline{\hat{\delta}} \tag{26}$$

So, the question posed here is: Can the proposed gain C_{RG} be found so that, in conjunction with the state reconstructor, the effects of $\underline{w}_2$ can be minimized?

For answers to these questions, a simulation is again used.

B. SIMULATION AND RESULTS

Figure 60 is a diagram of the composite plant-DAC system which was simulated on a digital computer. A listing of this simulation is shown in Appendix B.

The controller in this loop, with disturbance as shown, would probably be used only during the attitude controlled boost phase of this missile. However, for illustrative purposes, two of the time points shown in *Table 1* (9.85 sec, 135.8 sec) will be used for this investigation.

Figures 61 through *72* present the results obtained for the time points above. For each point, three runs are presented: (1) nominal run with no disturbance; (2) run with w_2 equal to the plant steady state response (x_1) due to the input command, from (1), but with $C_{RG} = 0.$; (3) same as (2) except C_{RG} is given an appropriate value. This value is determined from the ratio $(\delta/x_1)_{ss}$ from the undisturbed case, as would be expected.

For the 9.85 sec point, $C_{RG} = -3.54$, and it can be seen from comparison of *Figures 63* and *65* that the effects of w_2 are largely removed. For the t = 135.8 sec point, $C_{RG} = -1.45$, and similar conclusions are reached (compare *Figures 69* and *71*).

The gain matrix components for the DAC in the two cases above are given in *Table 5* where the eigenvalues of matrix $\underline{\underline{\tilde{B}}}$ were taken to be $\lambda_1, = -3., \lambda_2 = -5., \lambda_3 = -4. + j1., \lambda_4 = -4. - j1.$

TABLE 5. DAC GAIN COMPONENTS

GAIN VALUE \ TIME POINT (SEC)	9.85	135.8
k_{11}	101.18	29.55
k_{21}	-726.94	72.5
k_{12}	8.11	5.47
k_{22}	7.62	4.59

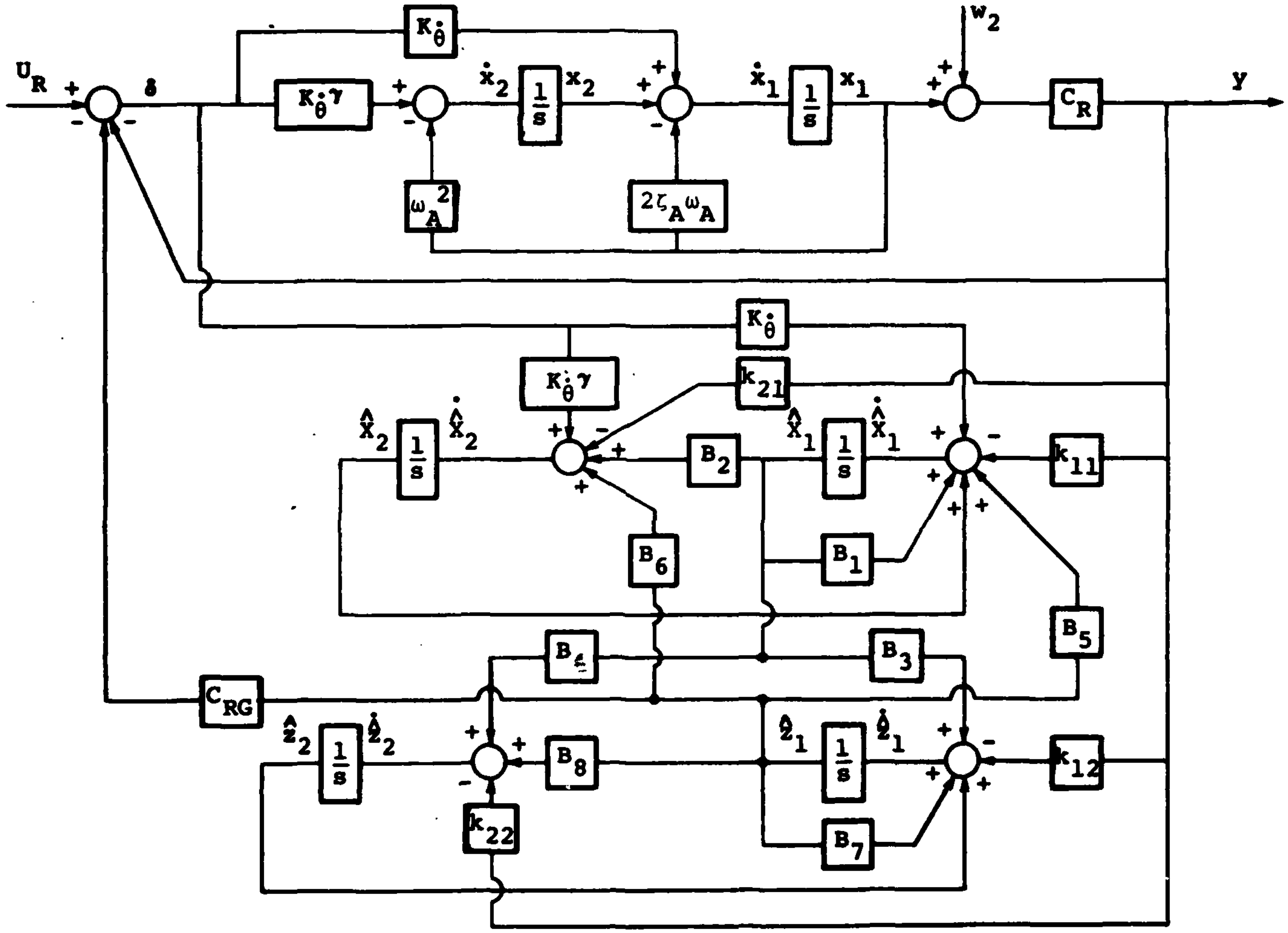


Figure 60. Plant-DAC composite diagram for rate loop with disturbance at output.

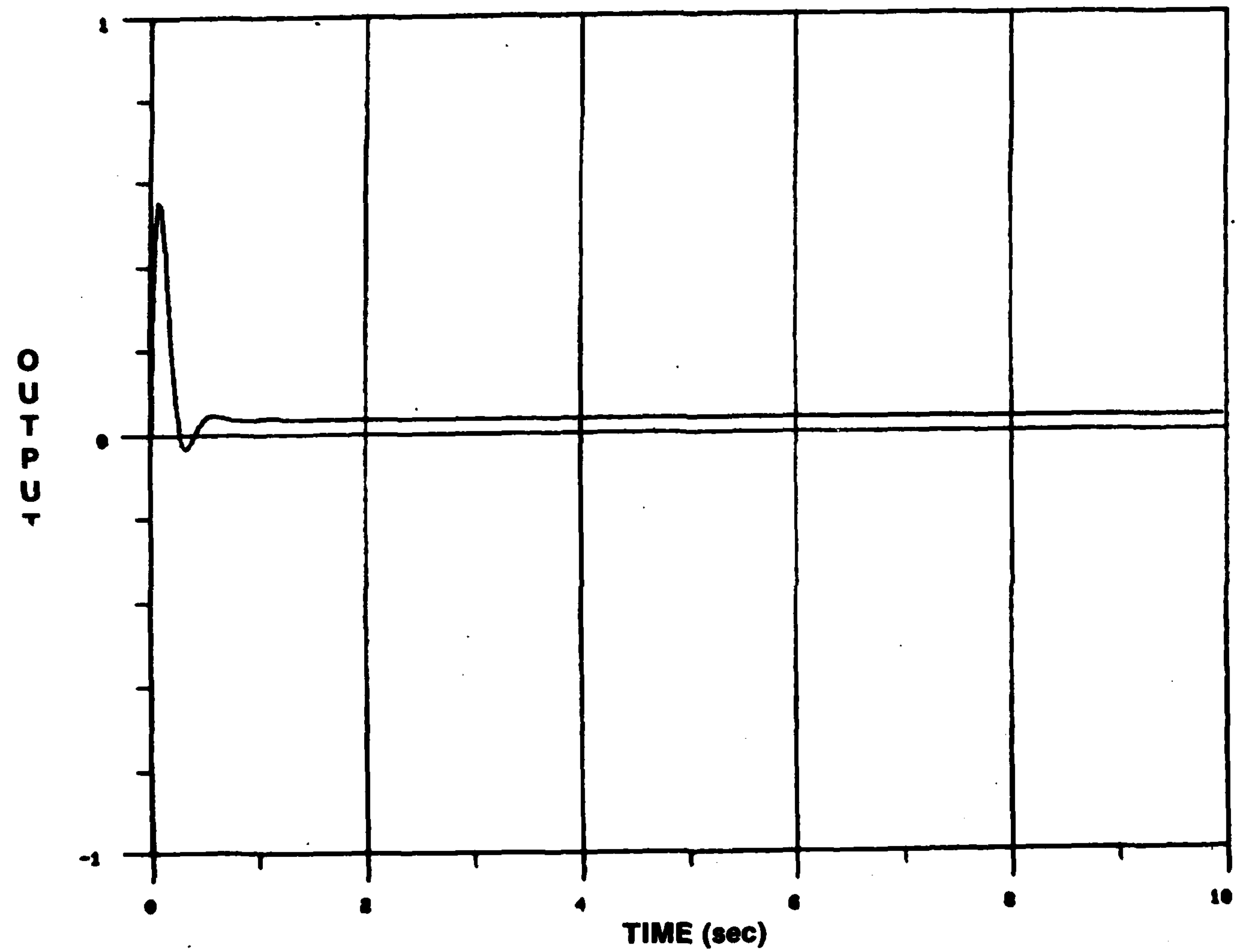


Figure 61. Rate loop response, t_f = 9.85 sec, U_R = 1, W_2 = 0.

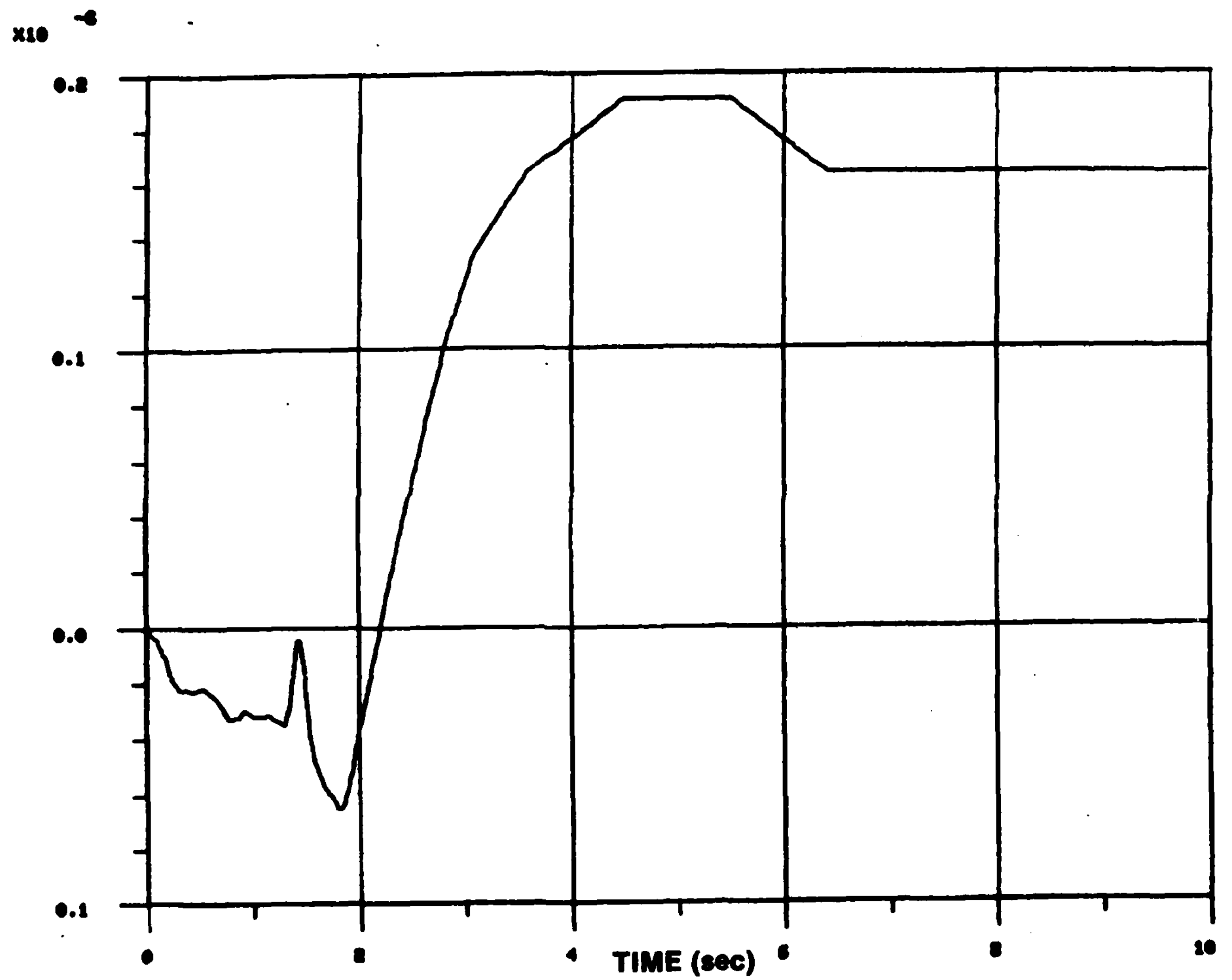


Figure 62. DAC disturbance estimation error, t_f = 9.85 sec, U_R = 1, W_2 = 0.

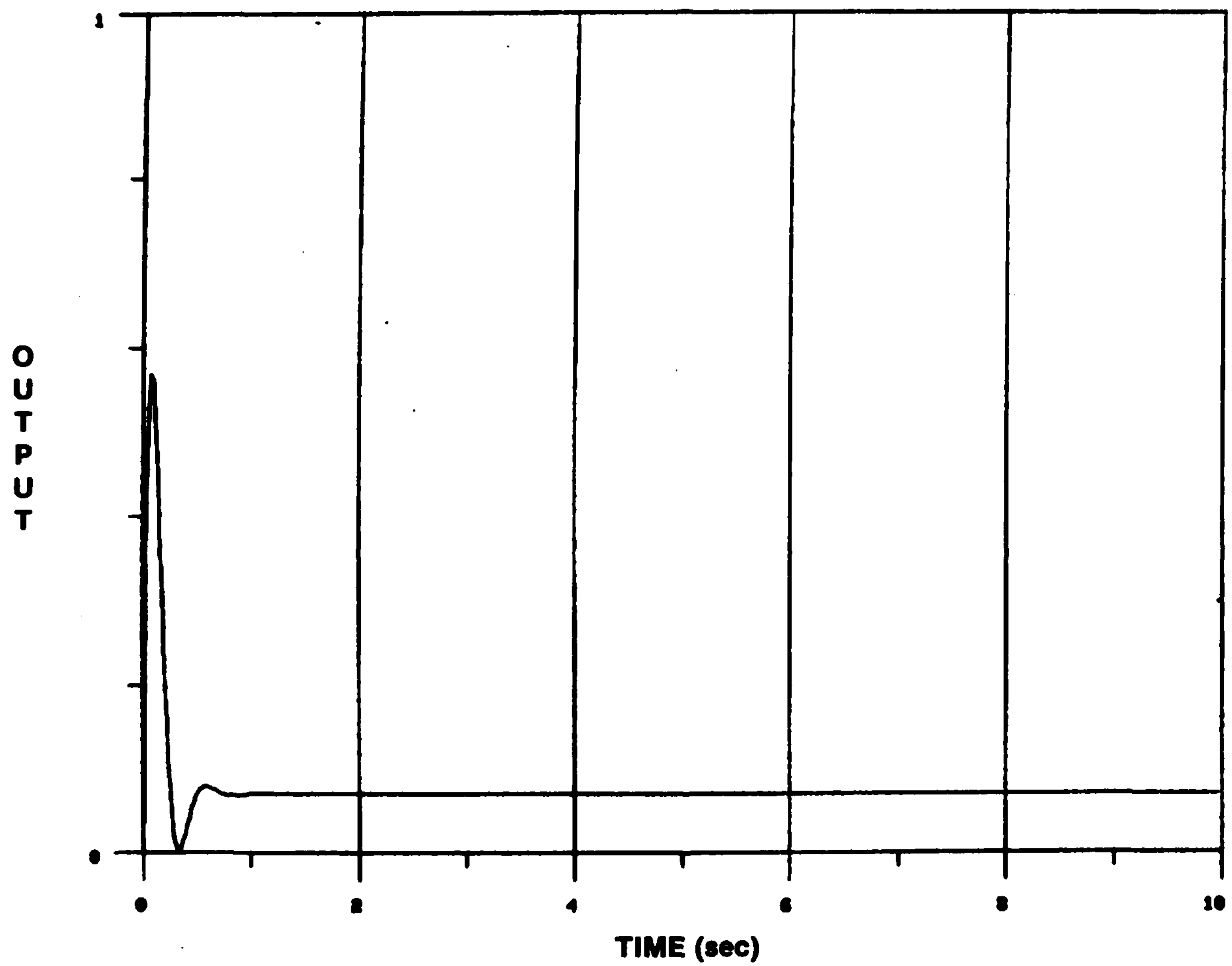


Figure 63. Rate loop response, $t_f = 9.85$ sec, $U_R = 1$, $W_2 = 0.26$, $C_{RG} = 0$.

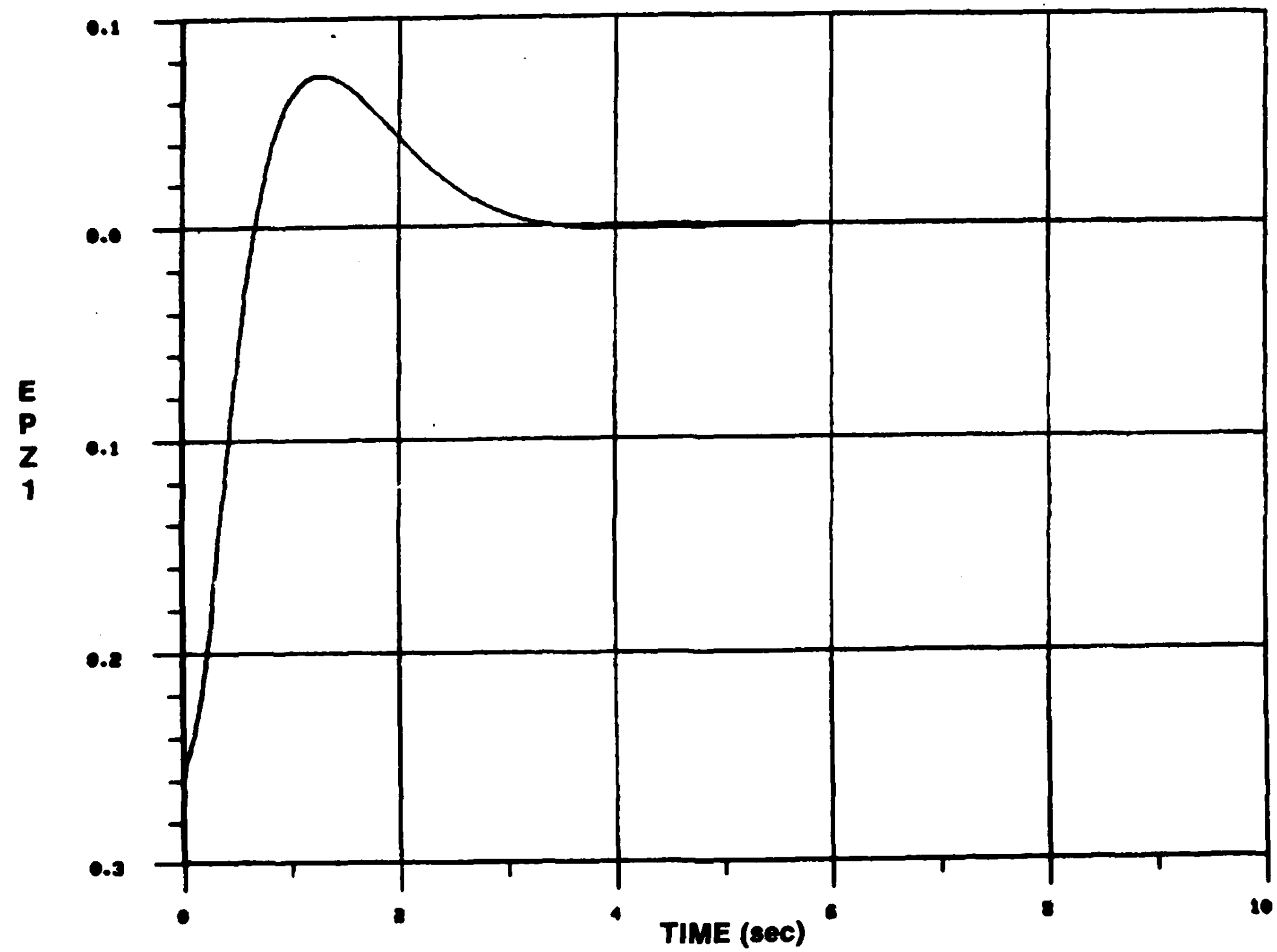


Figure 64. DAC disturbance estimation error, $t_f = 9.85$, $C_{RG} = 0$, $U_R = 1$, $W_2 = -0.26$.

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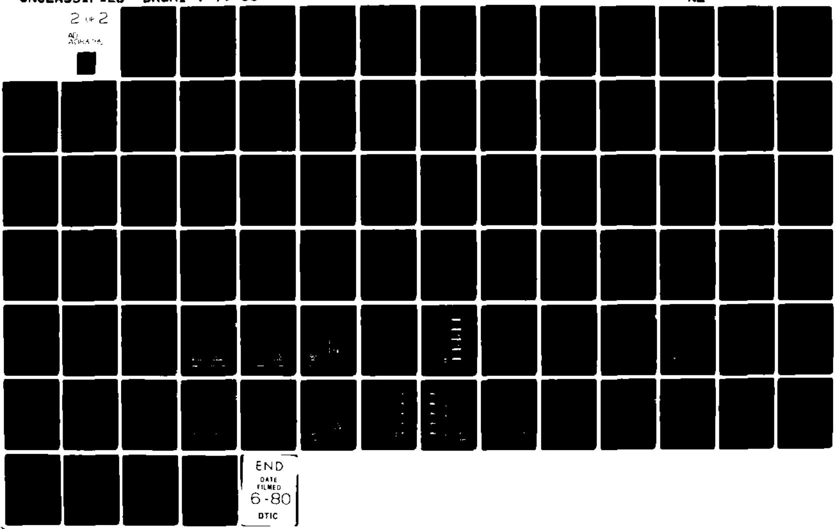

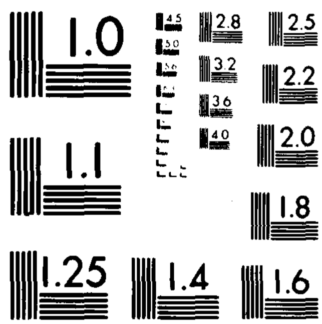

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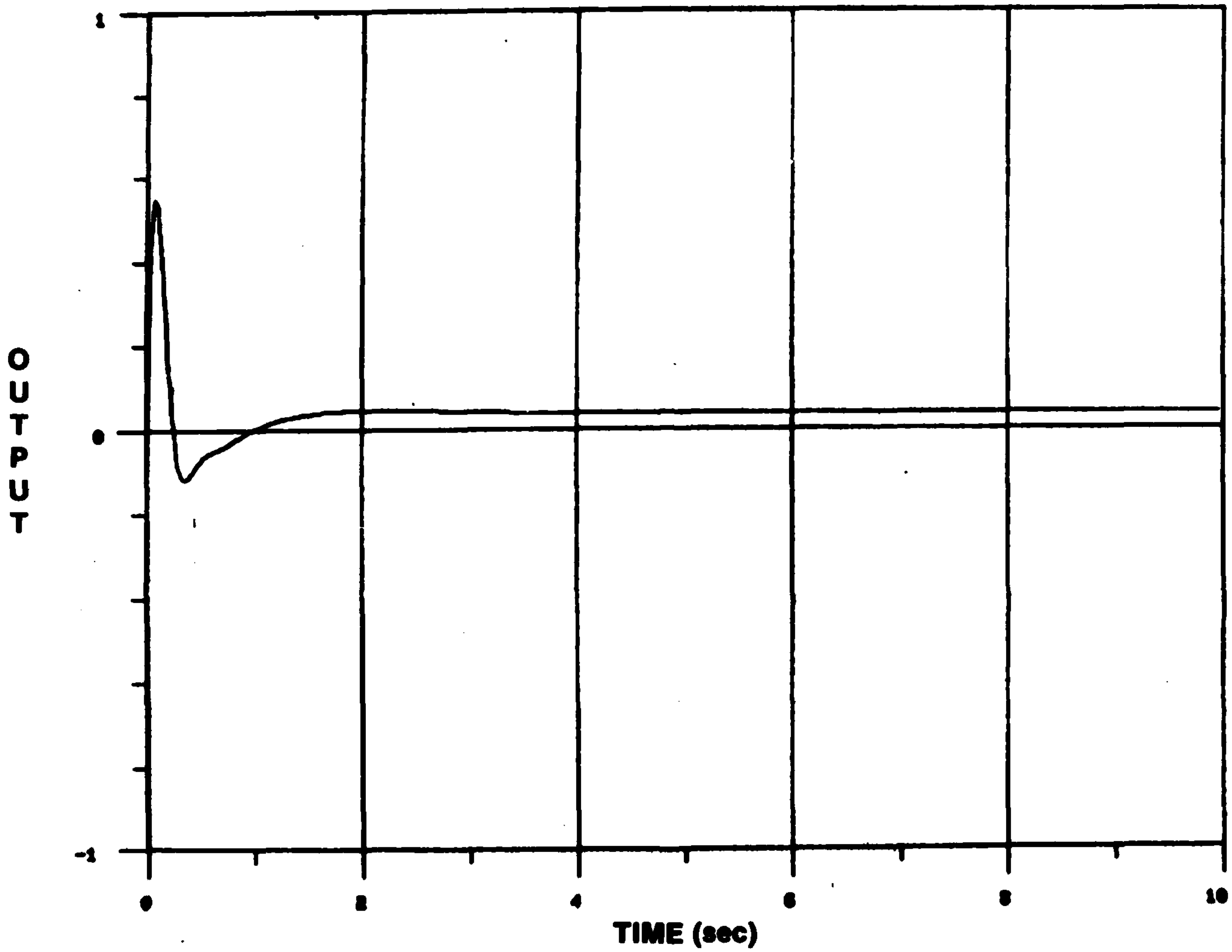

Figure 65. Rate loop response, t_f = 9.85 sec, U_R = 1, W_2 = -0.26, C_{RG} = -3.54.

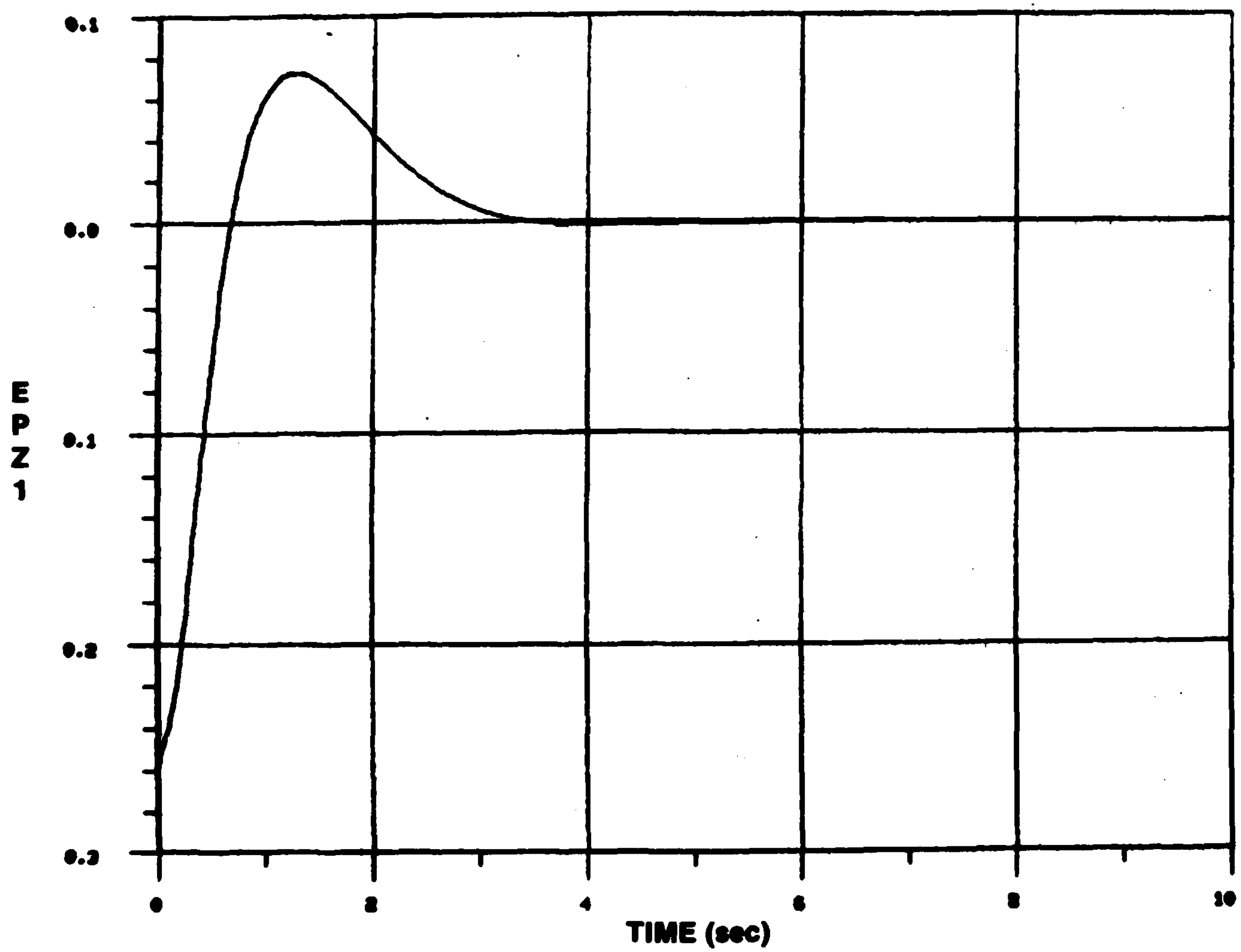

Figure 66. DAC disturbance estimation error, t_f = 9.85 sec, U_R = 1, W_2 = -0.26, C_{RG} = -3.54.

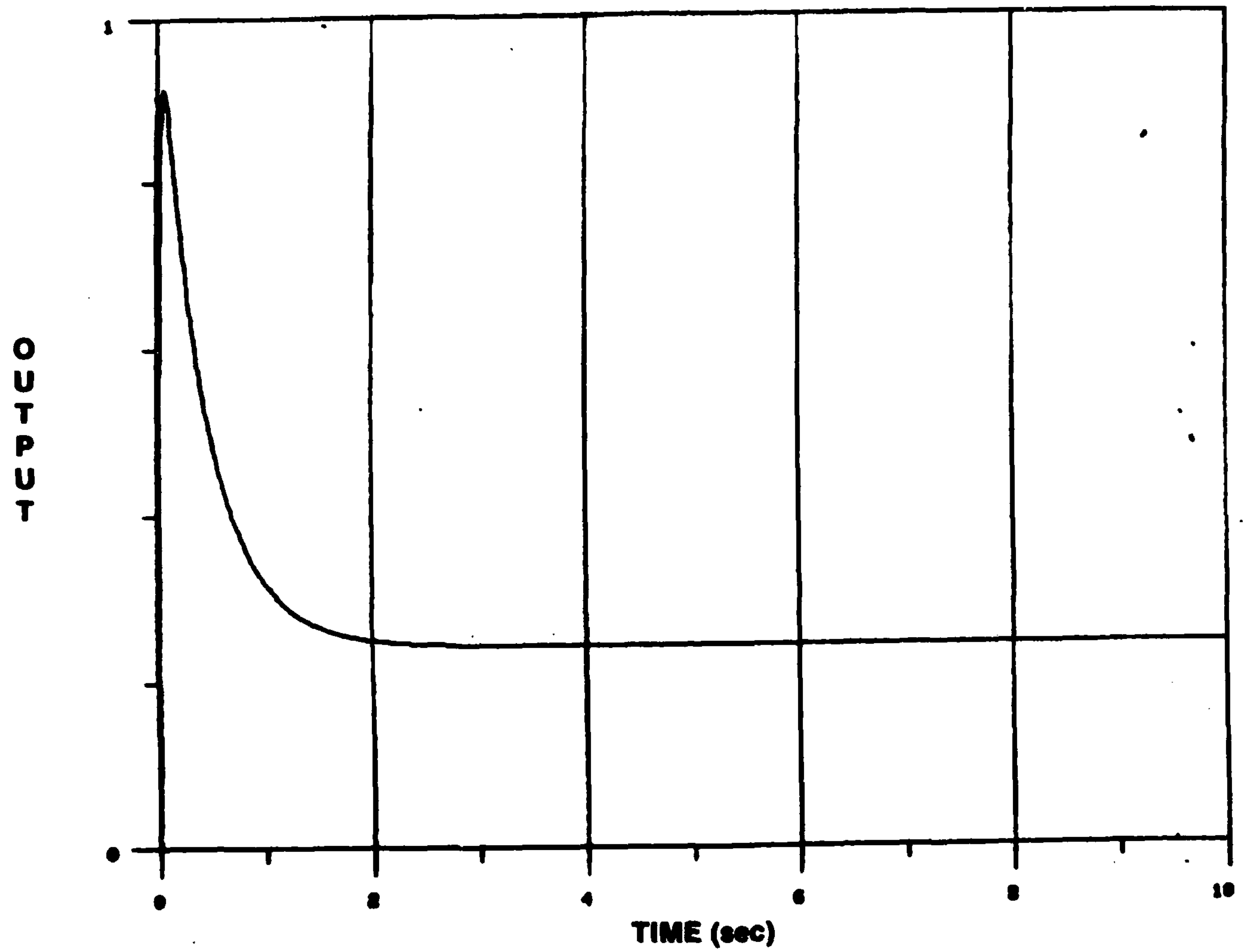

Figure 67. Rate loop response, t_f = 135.8 sec, U_R = 1, W_2 = 0.

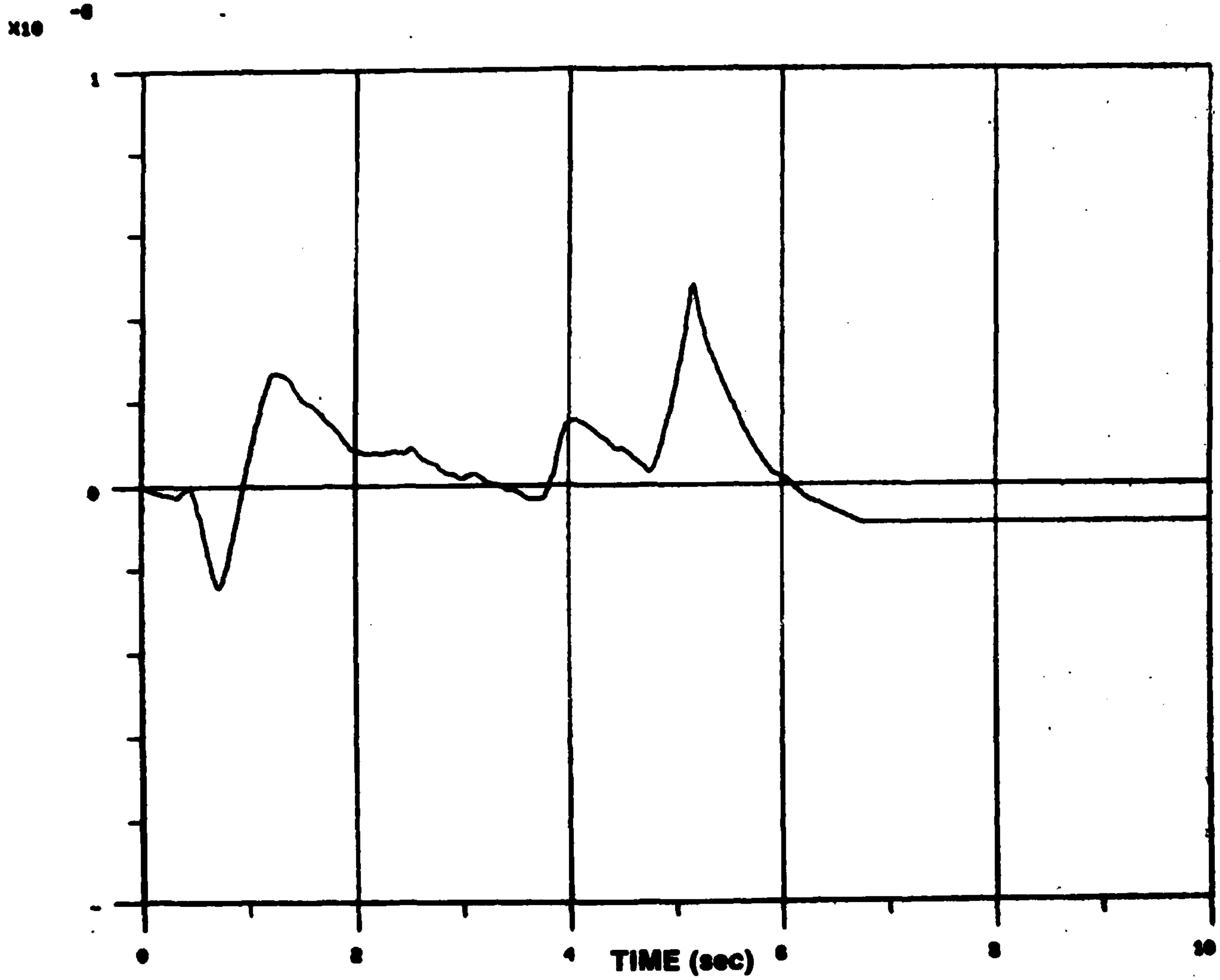

Figure 68. DAC disturbance estimation error, $t_f = 135.8$ sec, $U_R = 1$, $\overline{W}_2 = 8$.

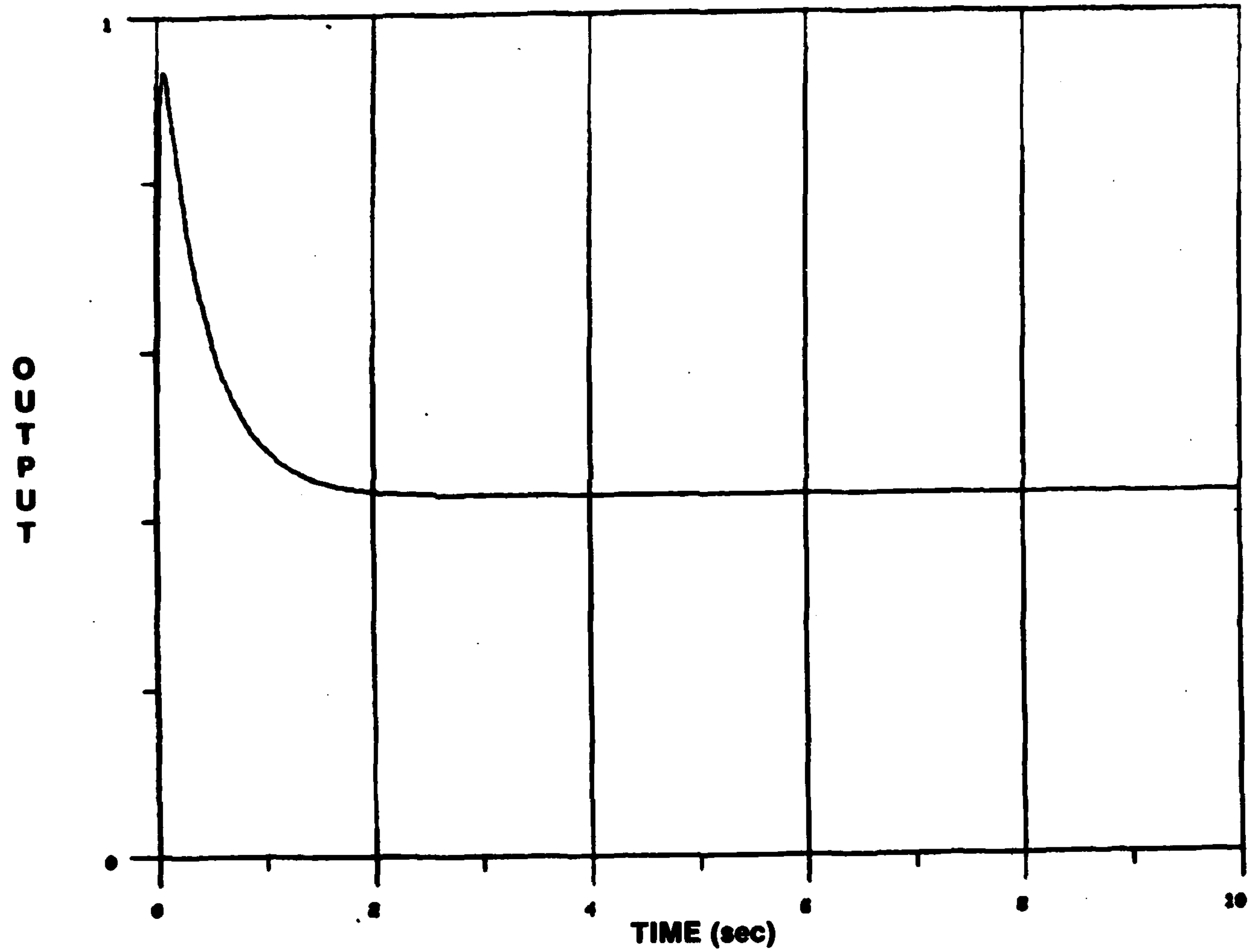

Figure 69. Rate loop response, $t_f = 135.8$ sec, $U_R = 1$, $W_2 = -0.5588$, $C_{RG} = 0$.

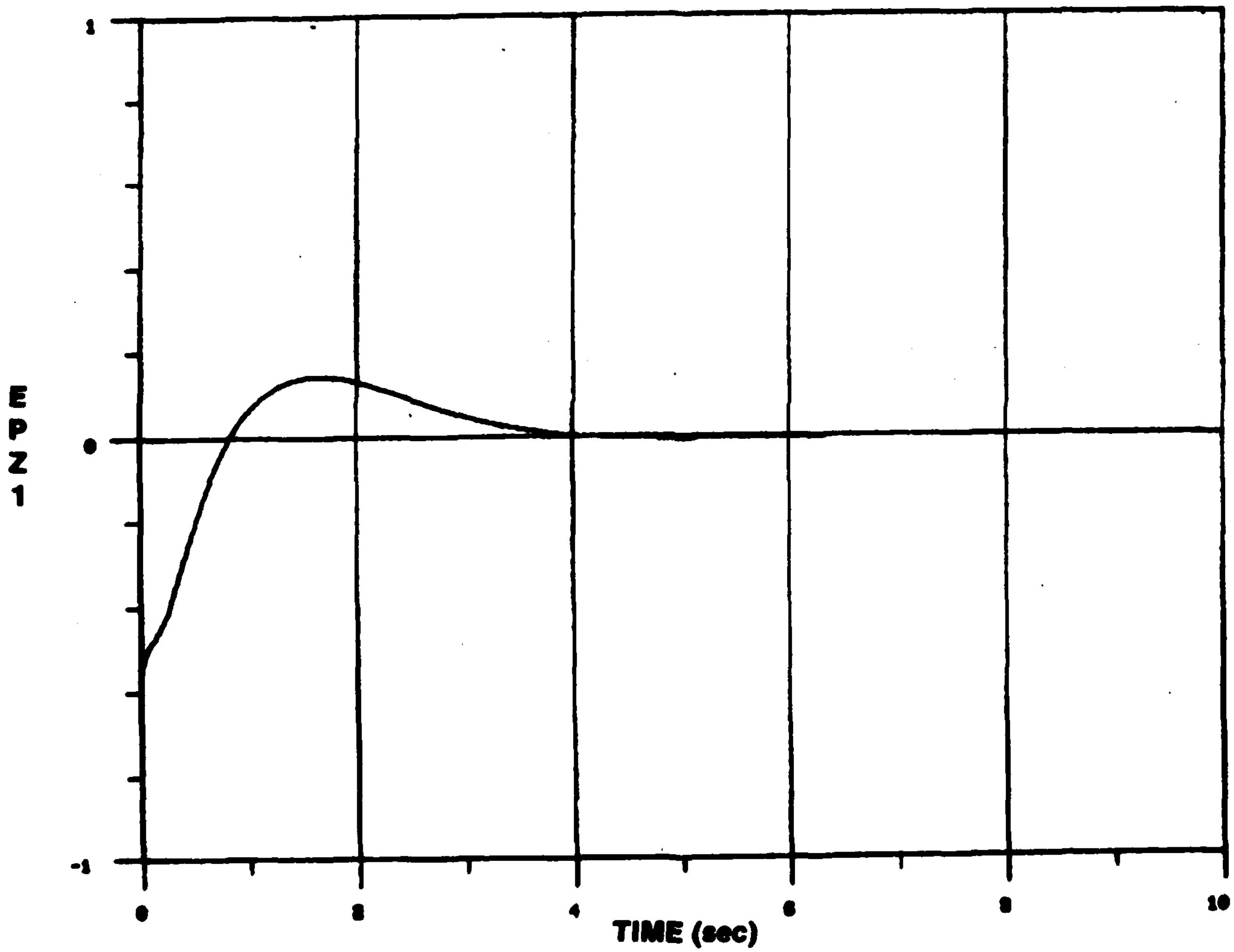

Figure 70. DAC disturbance estimation error, t_f = 135.8 sec, U_R = 1, W_2 = -0.5588, C_{RG} = 0.

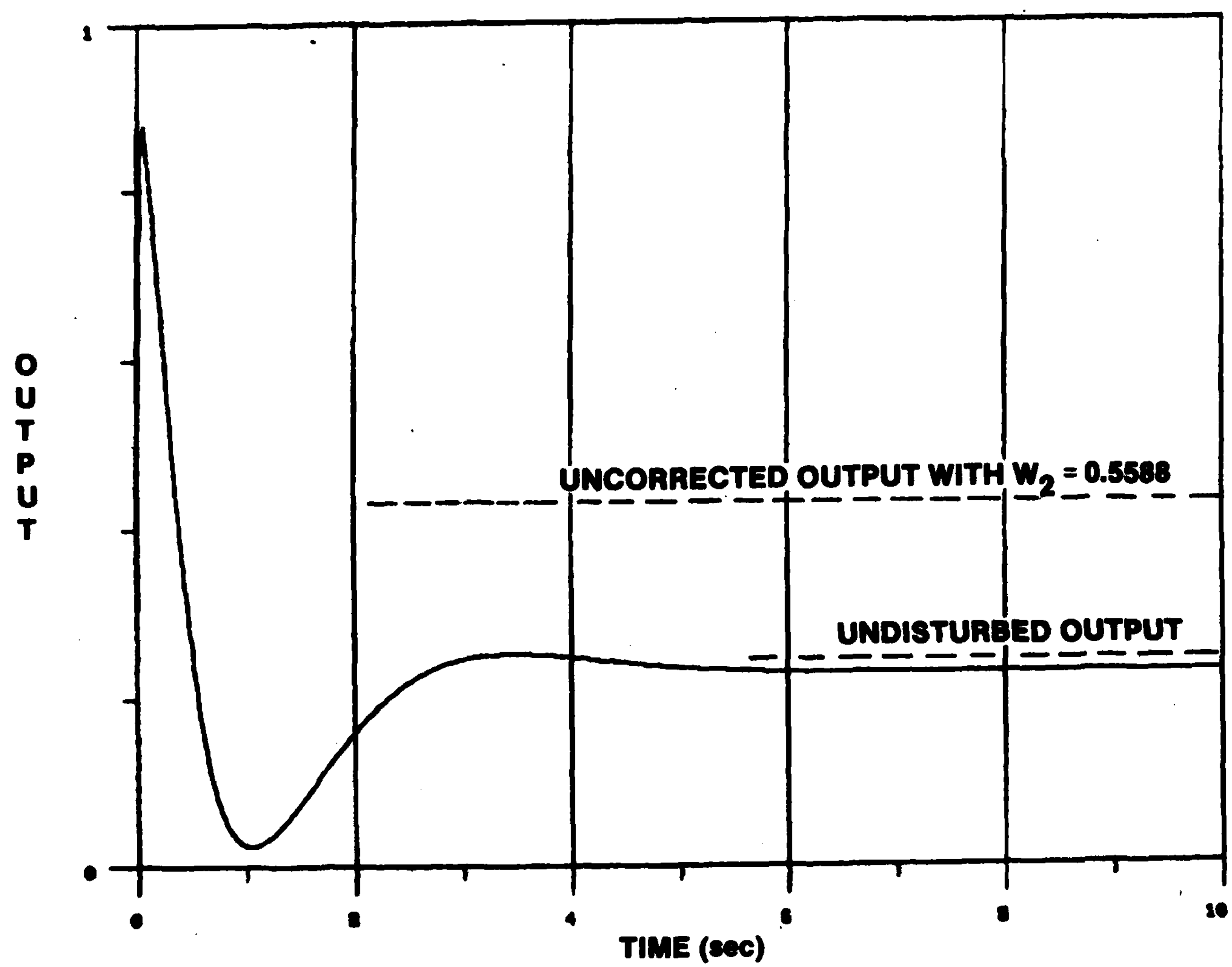

Figure 71. Rate loop response, t_f = 135.8 sec, U_R = 1, W_2 = -0.5588, C_{RG} = -1.45.

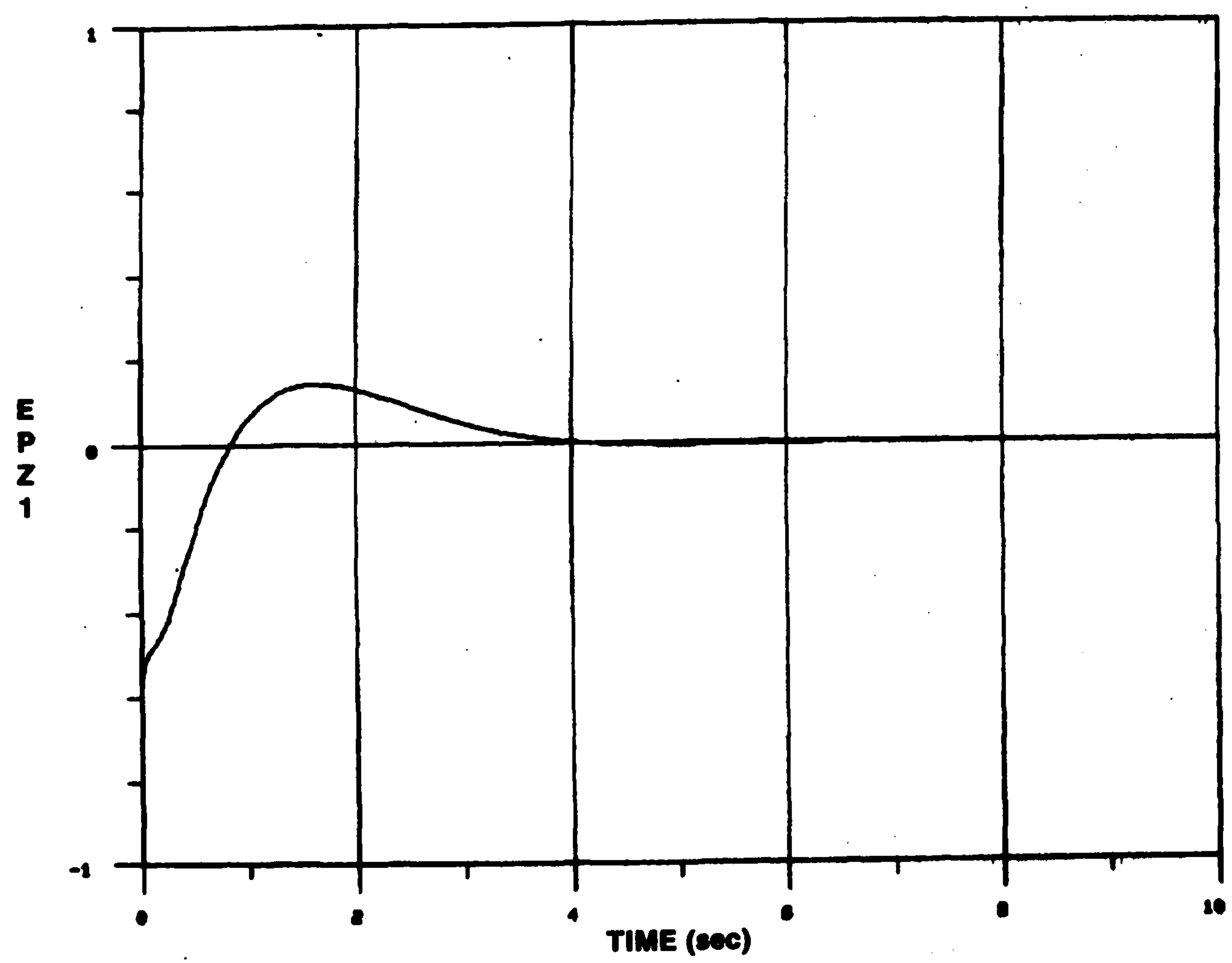

Figure 72. DAC disturbance estimation error, t_f = 135.8 sec, U_R = 1, W_2 = -0.5588, C_{RG} = -1.45.

C. CONCLUSIONS

From the results obtained here, even though there was no total absorption control $\underline{u}_c$ which would exactly cancel $\underline{w}_2$, by using a state reconstructor to estimate the value of the disturbance an implementation was possible whereby the effects of w_2 could be minimized.

7. ACCELERATION LOOP WITH DISTURBANCE ON OUTPUT

A. DAC MODEL DEVELOPMENT

As with the rate loop development in Section 6, this section will consider again a disturbance summed with the output, this time for the acceleration loop. This case is of much interest since the acceleration autopilot is used in the control loop from burnout onwards. A block diagram of this loop is shown in *Figure 73*.

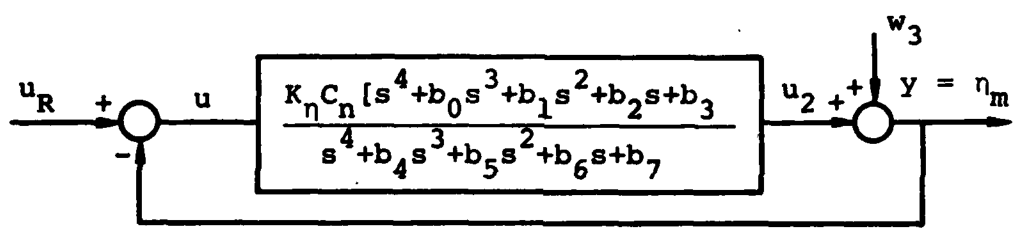

Figure 73. Acceleration loop with disturbance on output.

The transfer function from u to u_2 can be represented identically as shown in *Figure 3* with the same states and parameters. However, in this case, the matrix representation in the form of Equation (1) will be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -b_4 & 1 & 0 & 0 \\ -b_5 & 0 & 1 & 0 \\ -b_6 & 0 & 0 & 1 \\ -b_7 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + K_\eta C_n \begin{bmatrix} b_0-b_4 \\ b_1-b_5 \\ b_2-b_6 \\ b_3-b_7 \end{bmatrix} \underline{u} + [0]\ \underline{w}_3 \tag{27}$$

$$\underline{y} = [\,1\ 0\ 0\ 0\,] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [K_\eta C_n]\ \underline{u} + [1]\ \underline{w}_3 \ . \tag{28}$$

Using the same approach in this case as was used with the rate loop in the previous section, and for the same reason, a gain (C_{RAL}) is sought which can be used in conjunction with a disturbance state reconstructor output to accomplish a minimization of the effects of $\underline{w}_3$ on y.

Proceeding as before with the $\dot{\underline{\varepsilon}}$ dynamics one has

$$\dot{\underline{\varepsilon}} = \left[\begin{array}{c|c} \begin{bmatrix} -b_4 & 1 & 0 & 0 \\ -b_5 & 0 & 1 & 0 \\ -b_6 & 0 & 0 & 1 \\ -b_7 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \\ k_{41} \end{bmatrix} [1\ 0\ 0\ 0] & \begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \\ k_{41} \end{bmatrix} [1]\,[1\ 0] \\ \hline \begin{bmatrix} k_{12} \\ k_{22} \end{bmatrix} [1\ 0\ 0\ 0] & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} k_{12} \\ k_{22} \end{bmatrix} [1\ 0] \end{array}\right] \underline{\varepsilon} + \begin{bmatrix} \underline{0} \\ \hline \underline{\sigma} \end{bmatrix}$$

which reduces to

$$\dot{\underline{\varepsilon}} = \begin{bmatrix} (k_{11}-b_4) & 1 & 0 & 0 & k_{11} & 0 \\ (k_{21}-b_5) & 0 & 1 & 0 & k_{21} & 0 \\ (k_{31}-b_6) & 0 & 0 & 1 & k_{31} & 0 \\ (k_{41}-b_7) & 0 & 0 & 0 & k_{41} & 0 \\ k_{12} & 0 & 0 & 0 & k_{12} & 1 \\ k_{22} & 0 & 0 & 0 & k_{22} & 0 \end{bmatrix} \underline{\varepsilon} + \begin{bmatrix} \underline{0} \\ \underline{\sigma} \end{bmatrix} = \tilde{\underline{C}}\ \underline{\varepsilon} + \begin{bmatrix} \underline{0} \\ \underline{\sigma} \end{bmatrix}.$$

Solving for the eigenvalues of $\tilde{\underline{C}}$ gives

$$\lambda^6 + (b_4 - k_{11} - k_{12})\,\lambda^5 + (b_5 - b_4 k_{12} - k_{22} - k_{21})\,\lambda^4$$
$$+ (b_6 - b_4 k_{22} - b_5 k_{12} - k_{31})\,\lambda^3 + (b_7 - b_5 k_{22} - b_6 k_{12} - k_{41})\,\lambda^2$$
$$+ (-b_6 k_{22} - b_7 k_{12})\,\lambda - b_7 k_{22} = 0 \qquad (29)$$

In the same manner as for the first two cases, one can solve for the components of the gain matrices, $\underline{K}_1$ and $\underline{K}_2$. Doing so gives the following,

(a) $k_{11} = b_4 - k_{12} + A_0$

(b) $k_{21} = b_5 - b_4 k_{12} - k_{22} - A_1$

(c) $k_{31} = b_6 - b_4 k_{22} - b_5 k_{12} + A_2$

(d) $k_{41} = b_7 - b_5 k_{22} - b_6 k_{12} - A_3$

(e) $k_{12} = (b_6 k_{22} - A_4)\,/b_7$

(f) $k_{22} = -A_5/b_7$

where A_1 through A_5 are as defined for use in Equation (19). The full dimensional observer can now be expressed as

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \\ \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \end{bmatrix} = \begin{bmatrix} (k_{11}-b_4) & 1 & 0 & 0 & k_{11} & 0 \\ (k_{21}-b_5) & 0 & 1 & 0 & k_{21} & 0 \\ (k_{31}-b_6) & 0 & 0 & 1 & k_{31} & 0 \\ (k_{41}-b_7) & 0 & 0 & 0 & k_{41} & 0 \\ k_{12} & 0 & 0 & 0 & k_{12} & 1 \\ k_{22} & 0 & 0 & 0 & k_{22} & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} - \begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \\ k_{41} \\ k_{12} \\ k_{22} \end{bmatrix} \underline{y} + K_\eta C_n \begin{bmatrix} b_0-b_4+k_{11} \\ b_1-b_5+k_{21} \\ b_2-b_6+k_{31} \\ b_3-b_7+k_{41} \\ k_{12} \\ k_{22} \end{bmatrix} \underline{u} \qquad (30)$$

The question for this case is of the same type as for the rate loop, i.e., can a gain, C_{RAL}, be found and used in conjunction with the state reconstructor output $\hat{z}_1$, to minimize the effects of the disturbance?

B. SIMULATION AND RESULTS

The diagram for this composite system, with proposed control, is shown in *Figure 74*. The e's and h's are as defined for *Figure 3* with the following exceptions:

$$e_6 = k_{11}$$

$$e_7 = k_{21}$$

$$e_8 = k_{31}$$

$$e_9 = k_{41}$$

$$e_{10} = k_{12}$$

$$e_{11} = k_{22}$$

A listing of the digital simulation is given in Appendix C.

For this loop, the gain, C_{RAL}, is determined initially from the ratio $\frac{u}{y}$ (see *Figure 74*) from the undisturbed case and is then iterated, if necessary, to obtain a final value.

Several of the time points from *Table 1* were used to analyze this case. *Figures 75* through *84* give results for the 9.85 sec airframe parameters. By comparing *Figures 75, 77* and *79*, it can be seen that the effects of the disturbance are cancelled for a disturbance magnitude equal to the input command. *Figures 81* and *82* show results for w_3 equal to twice the input command magnitude and *Figures 83* and *84* are results for w_3 equal to five times the input command.

For t_f = 66.7 sec (apogee), an input command of 0.5 was used with w_3 = 0.5. *Figures 85* through *89* give the results for this time point. As can be seen from *Figures 85, 87* and *89*, the disturbance effects were again cancelled out although, since the system is sluggish, it takes longer to settle out than the previous case.

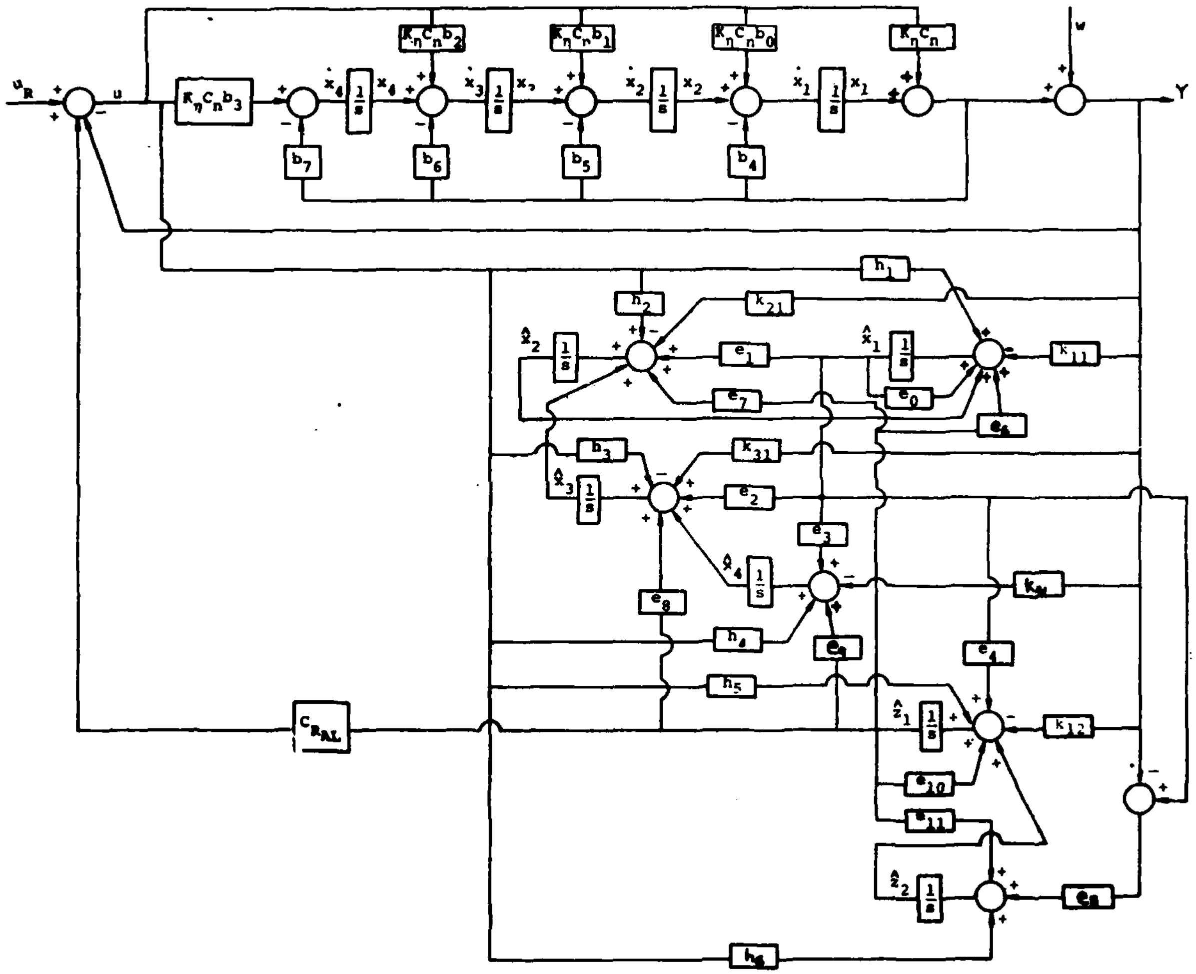

Figure 74. Plant-DAC composite for acceleration loop with disturbance at output.

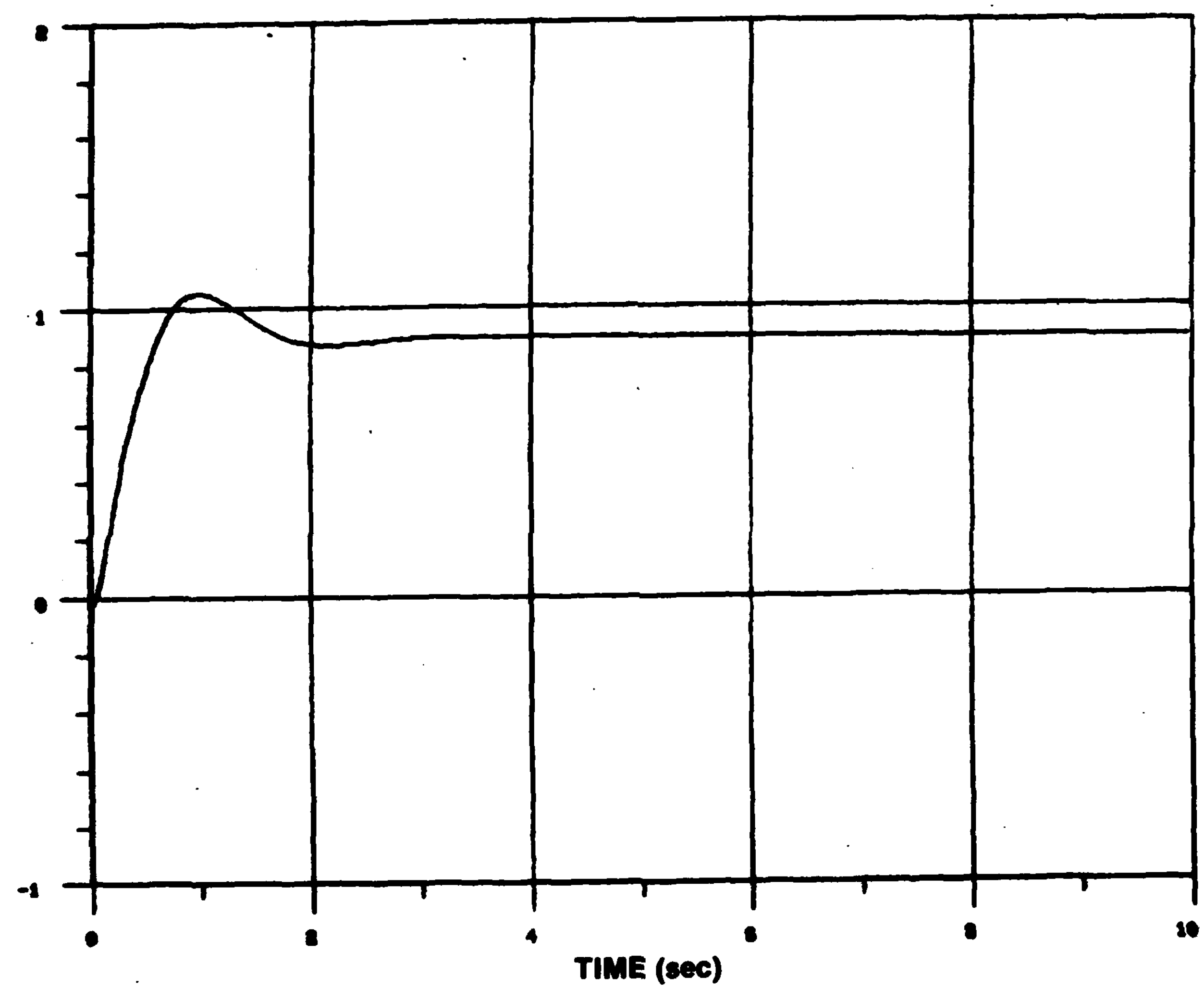

Figure 75. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_3 = 0.

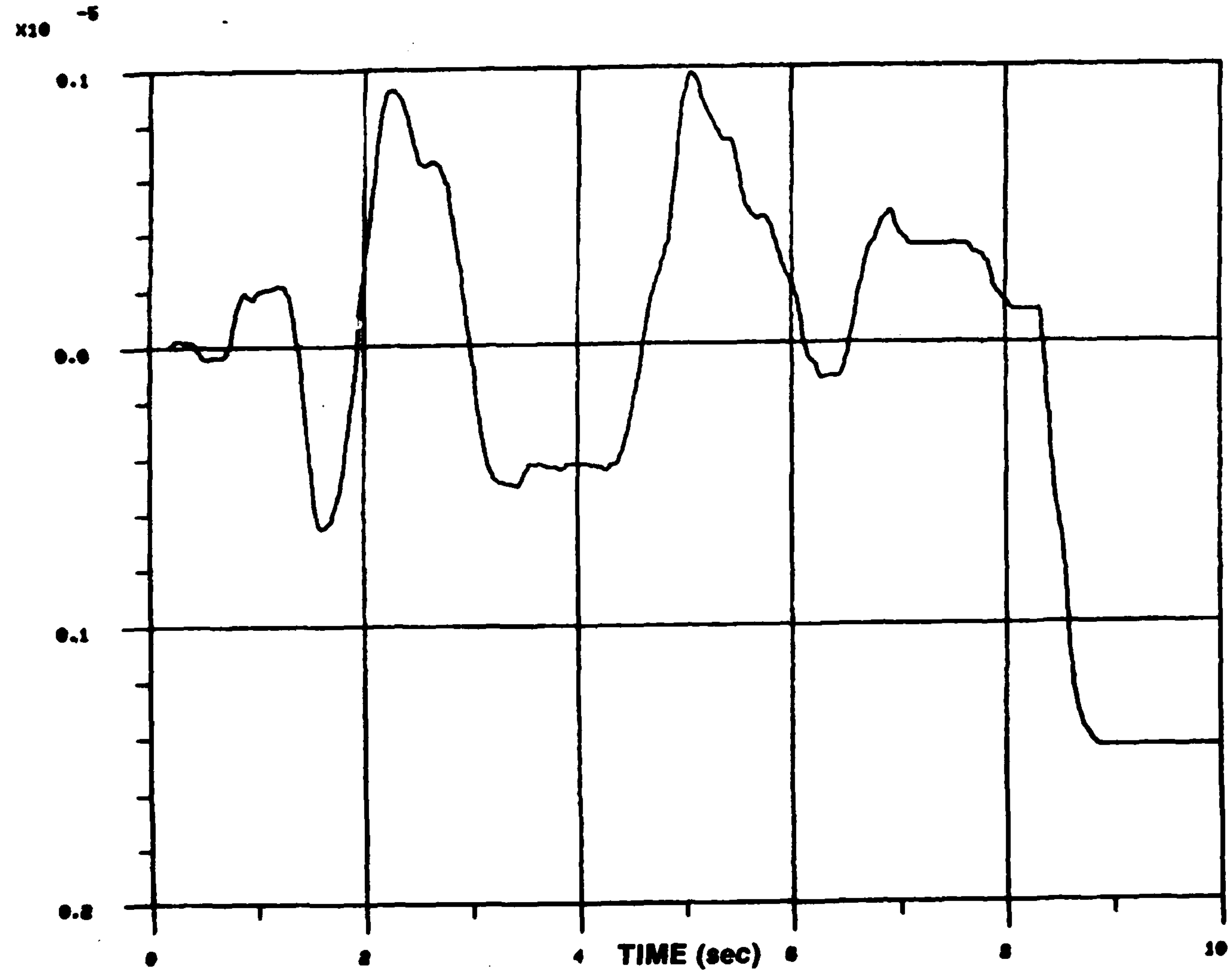

Figure 76. DAC disturbance estimation error, $t_f = 9.85$ sec, PGO = 1, $W_3 = 0$.

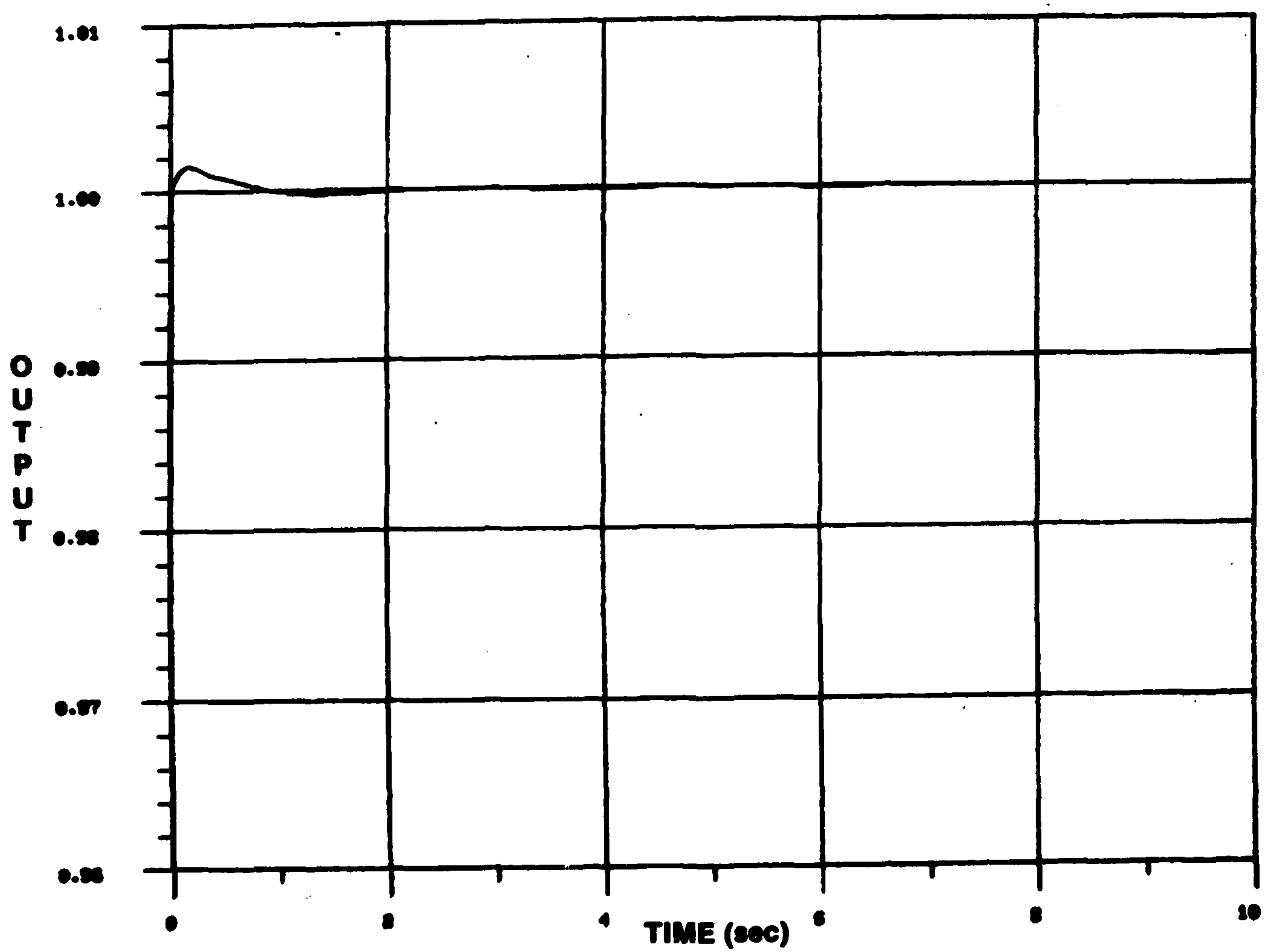

Figure 77. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_3 = 1, C_{RAL} = 0.

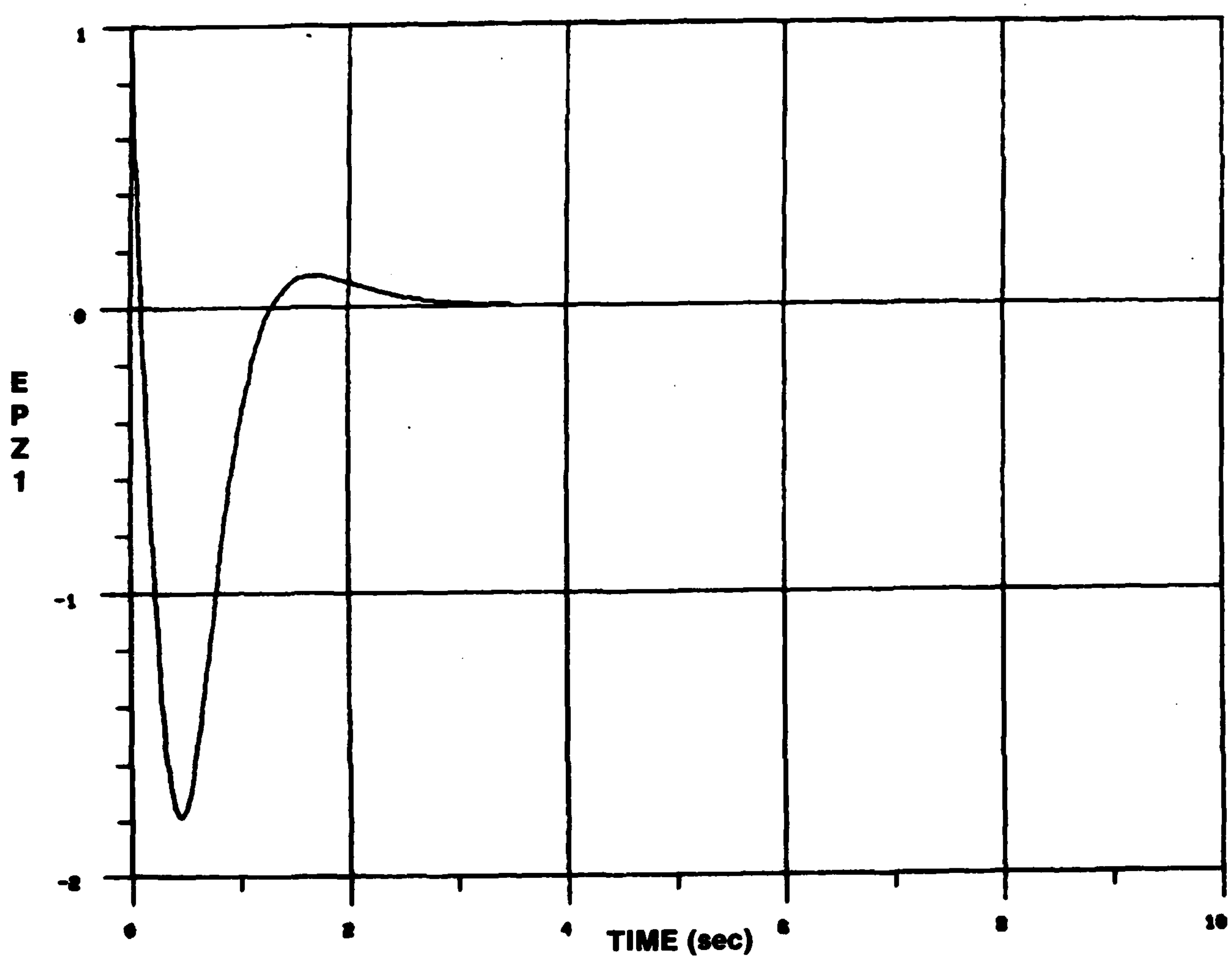

Figure 78. DAC disturbance estimation error, t_f = 9.85 sec, PGO = 1, W_3 = 1, C_{RAL} = 0.

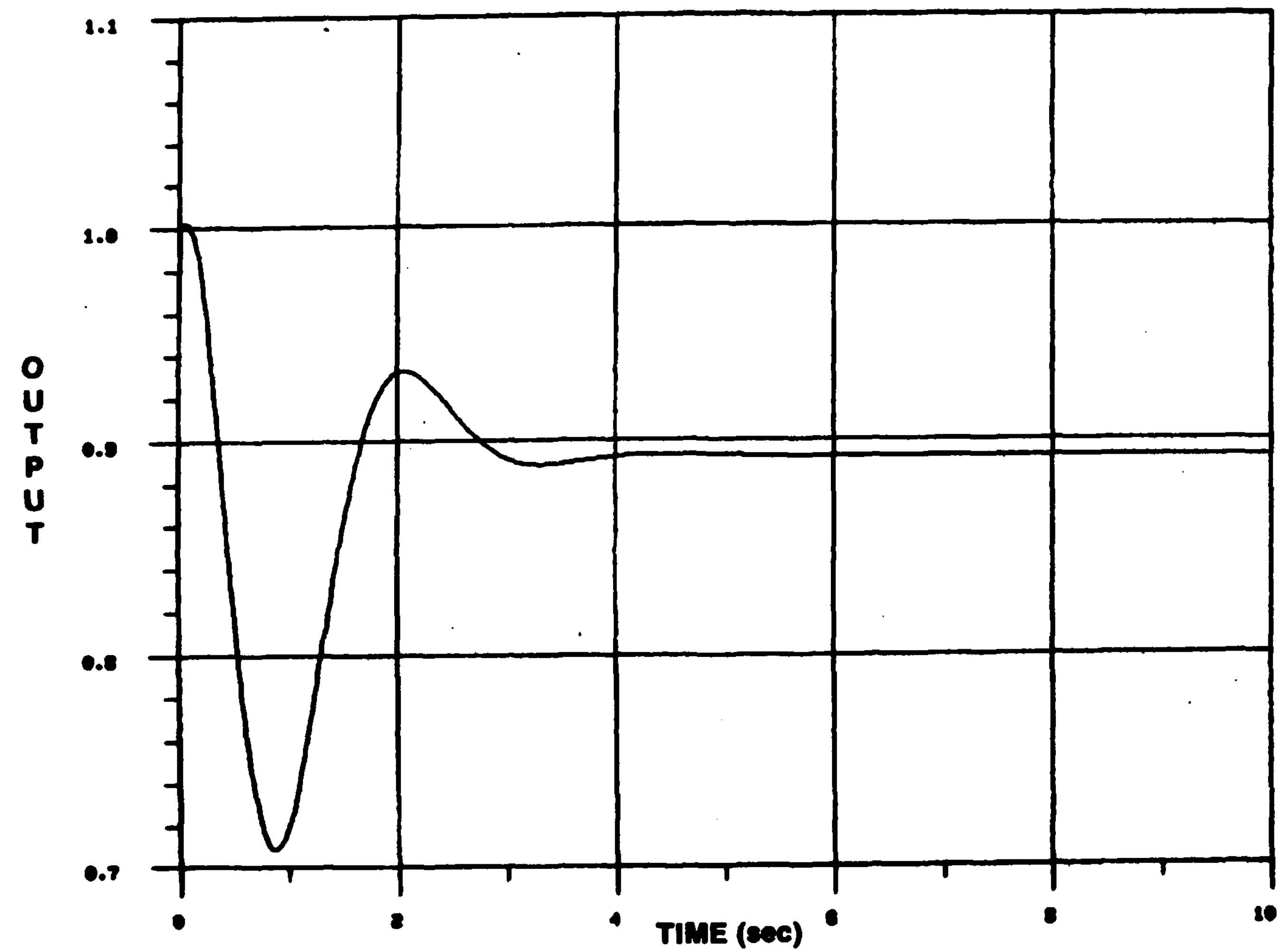

Figure 79. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_3 = 1, C_{RAL} = -0.12.

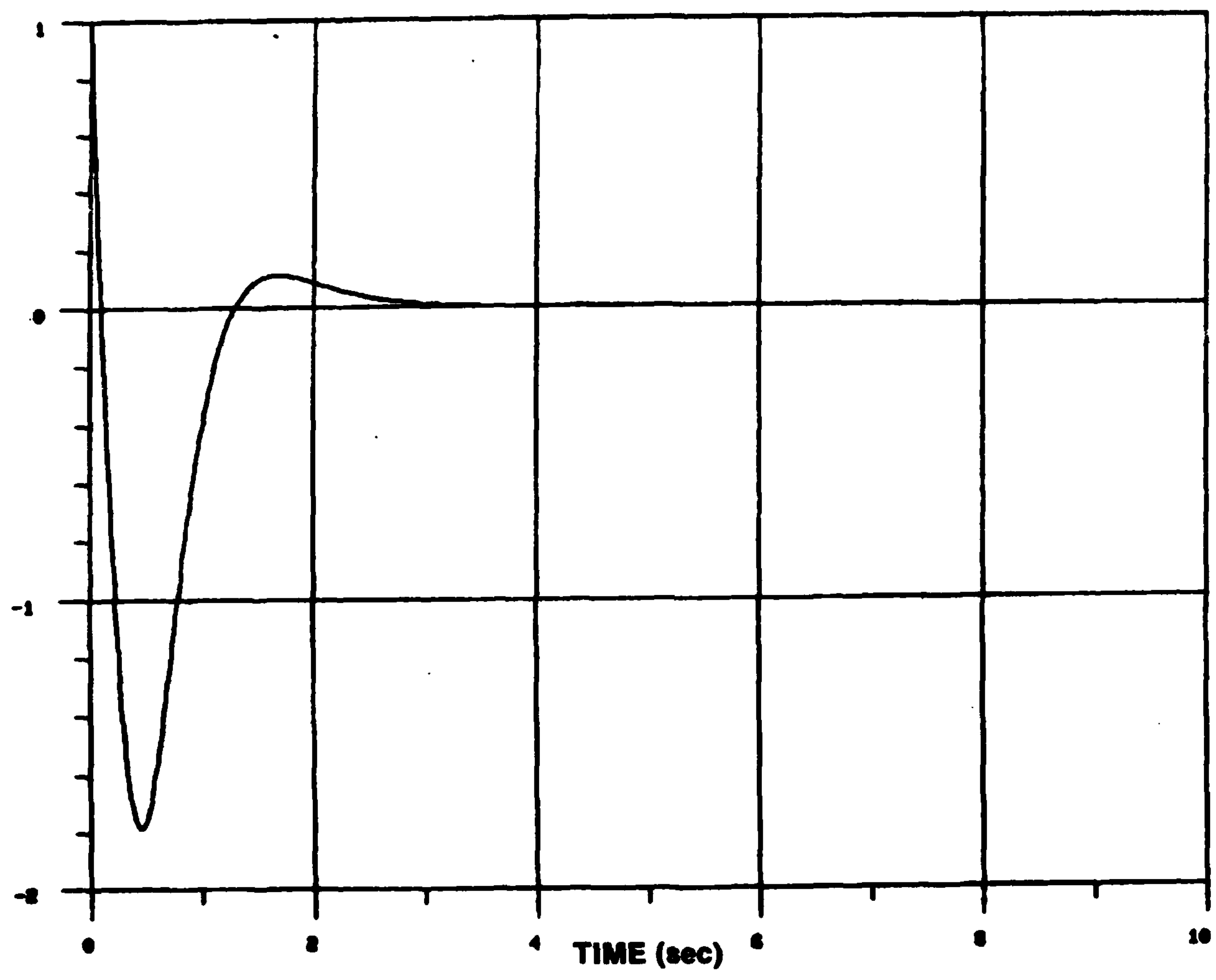

Figure 80. DAC disturbance estimation error, t_f = 9.85 sec, PGO = 1, W_3 = 1, C_{RAL} = -0.12.

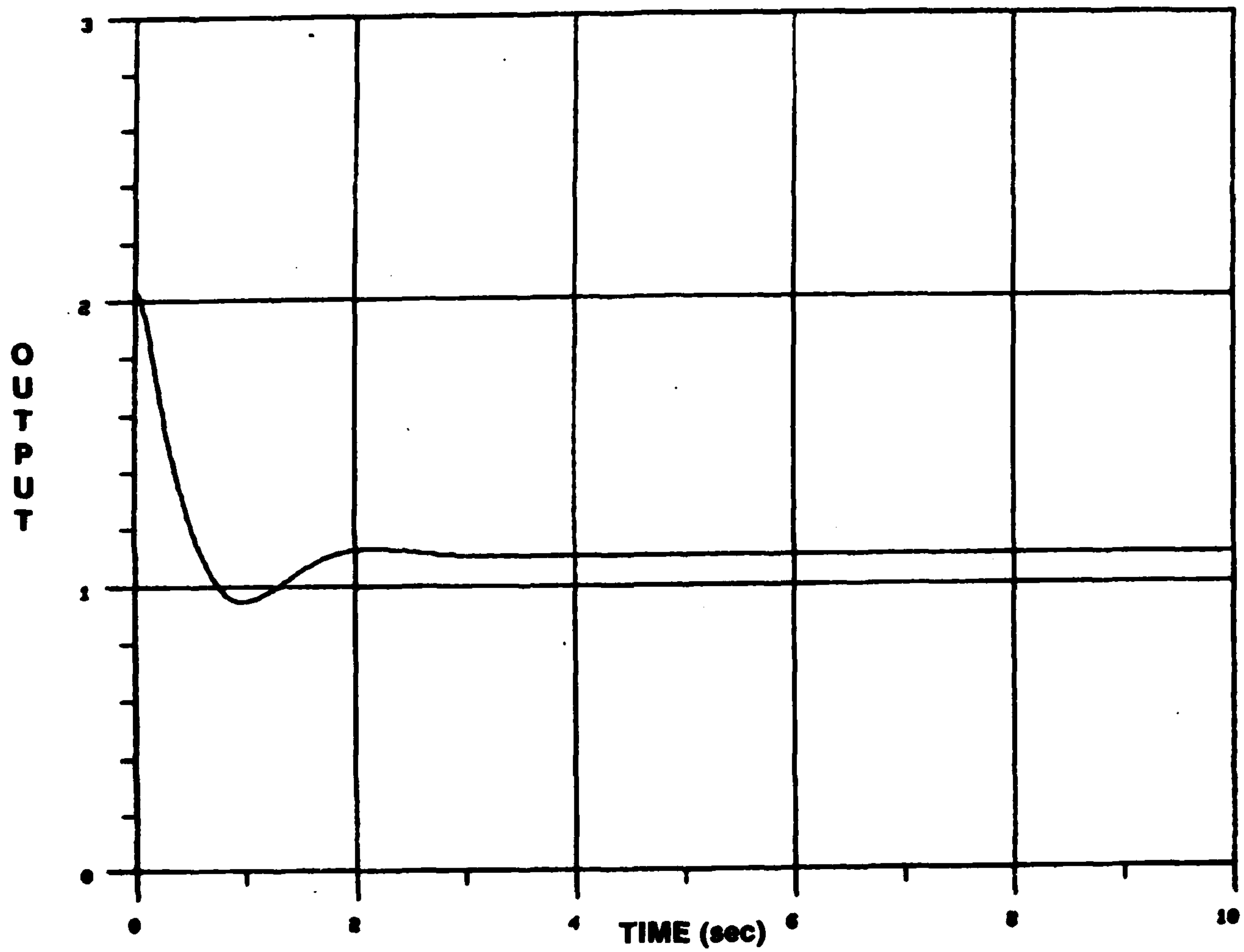

Figure 81. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_3 = 2, C_{RAL} = 0.

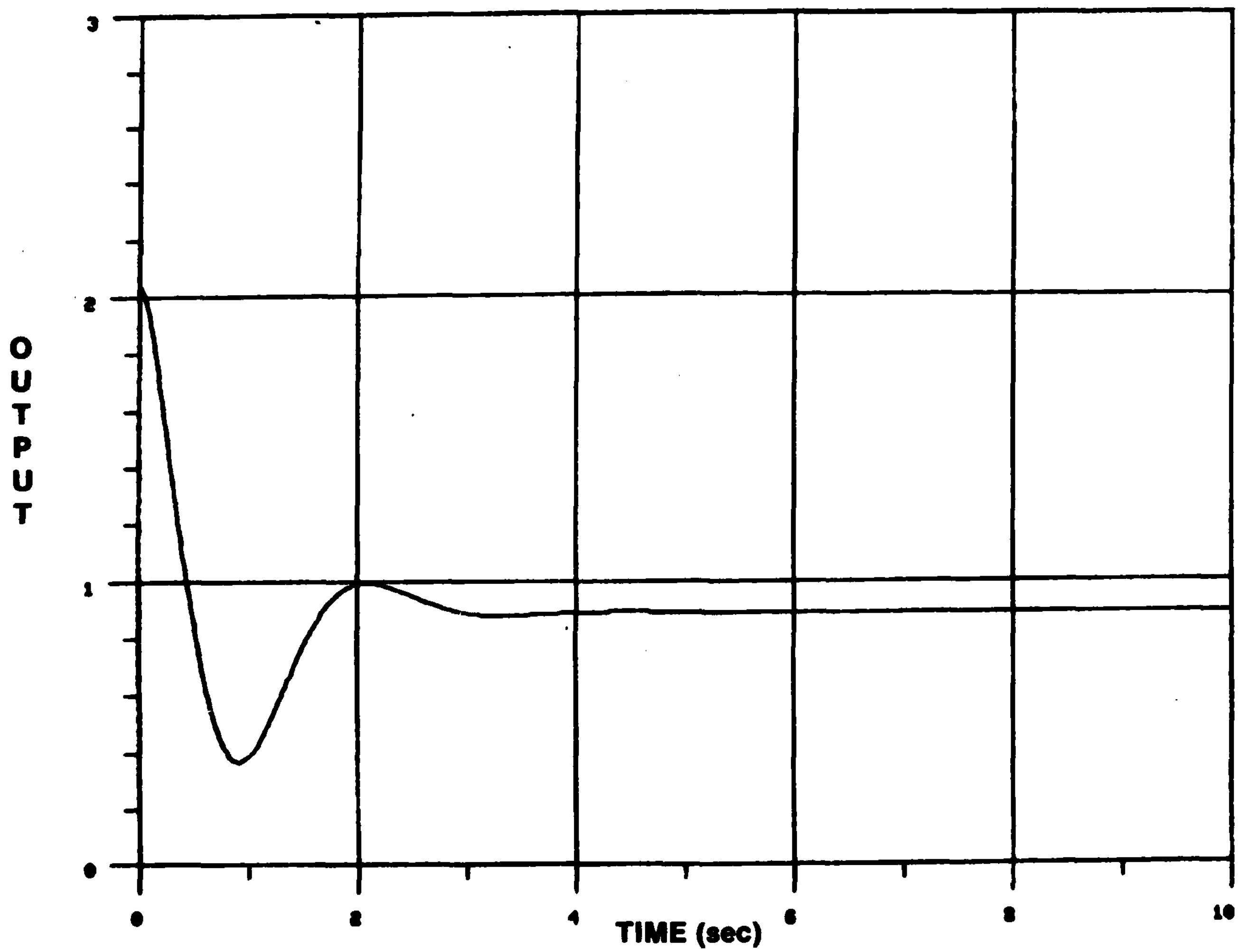

Figure 82. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_3 = 2, C_{RAL} = -0.12.

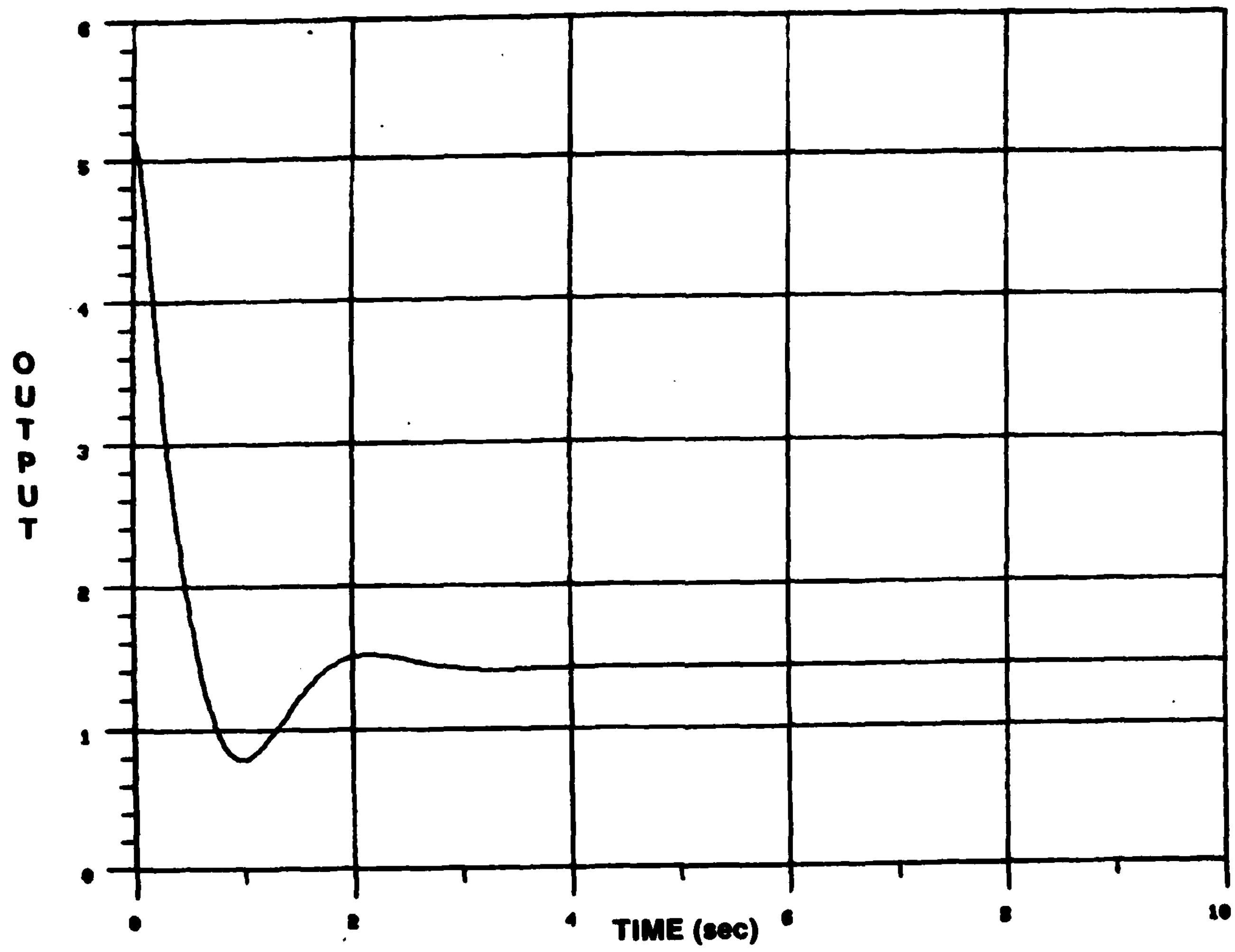

Figure 83. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_3 = 5, C_{RAL} = 0.

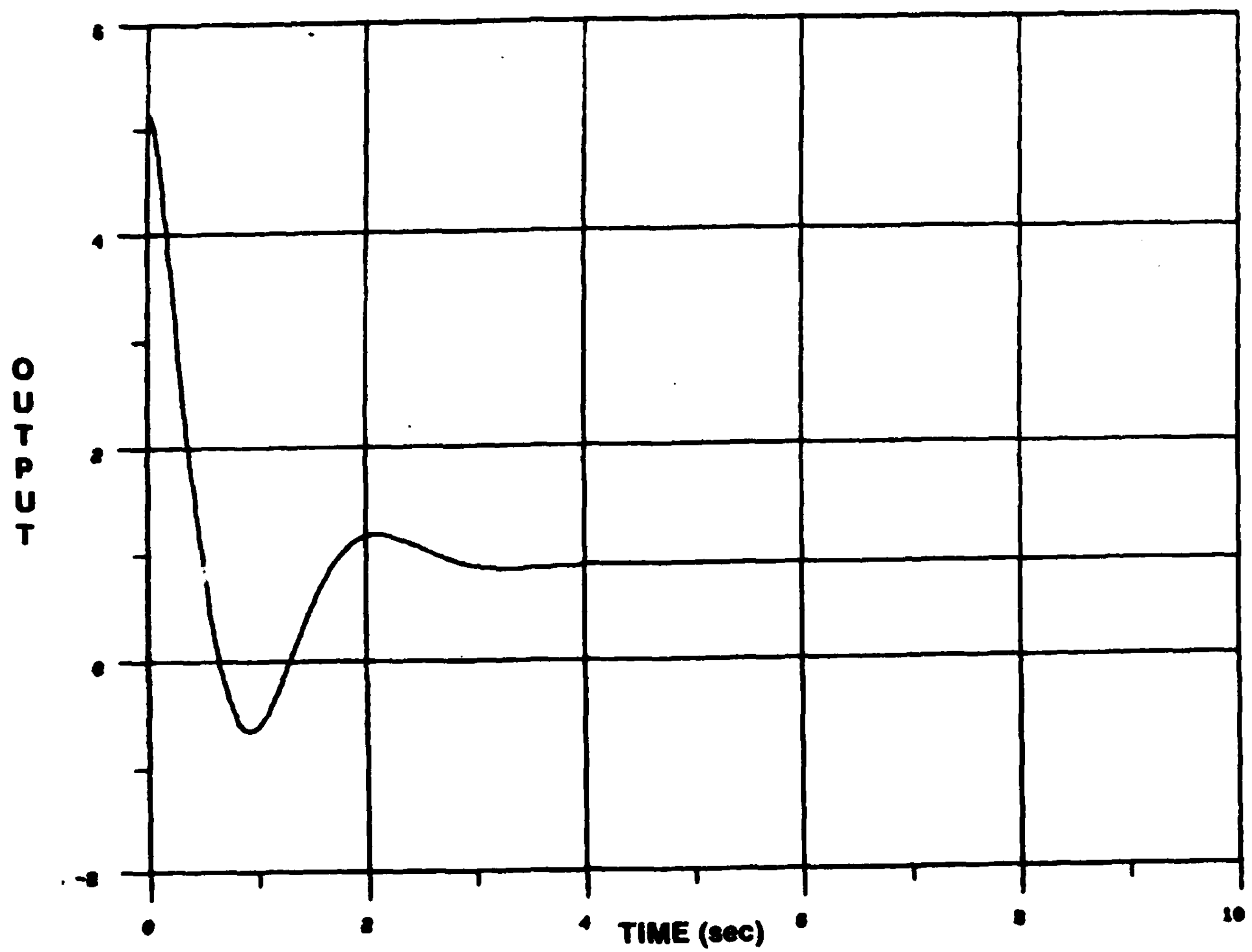

Figure 84. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_3 = 5, C_{RAL} = -0.12.

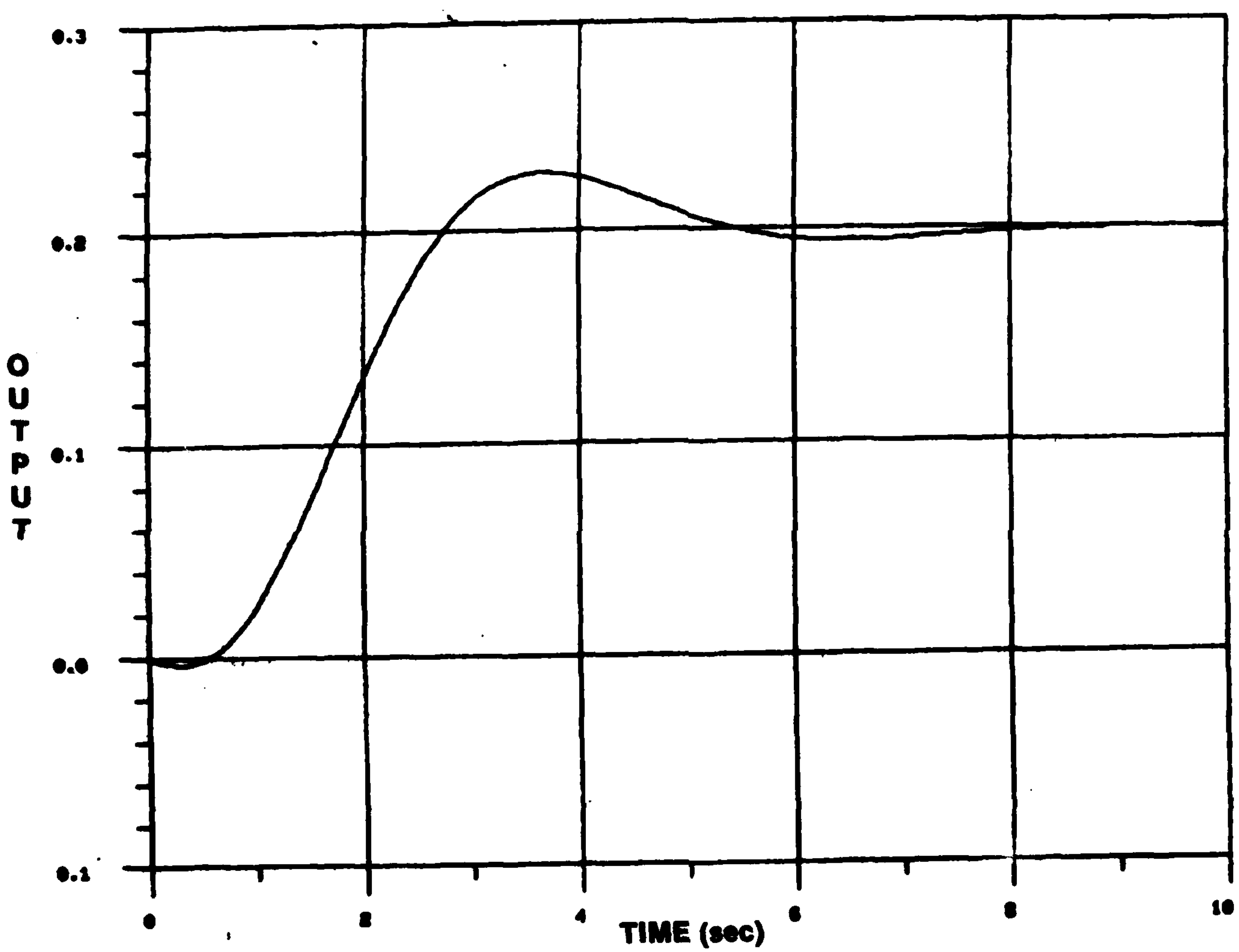

Figure 85. Acceleration loop response, t_f = 66.7 sec, PGO = 0.5, W_3 = 0.

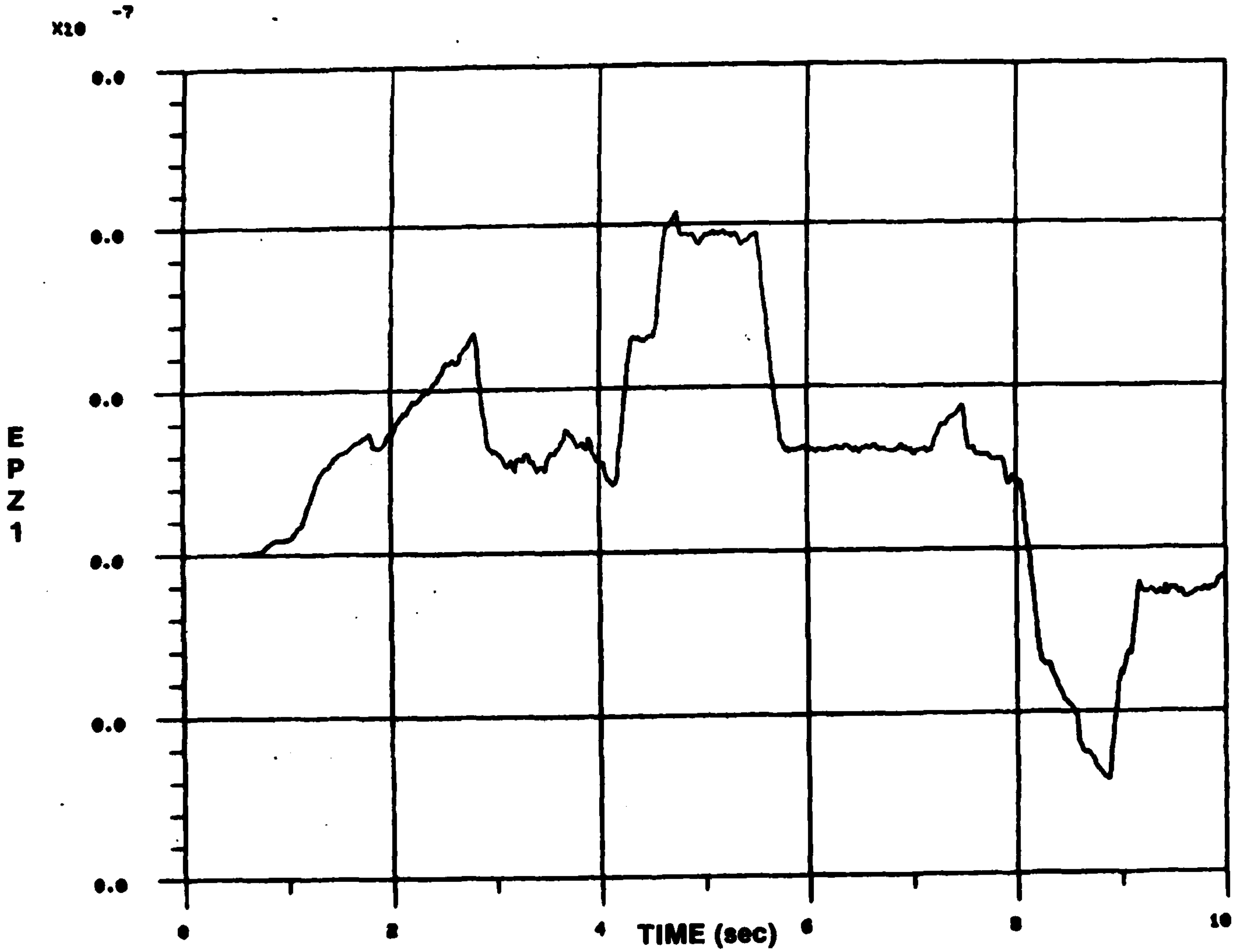

Figure 86. DAC disturbance estimation error, t_f = 66.7 sec, PGO = 0.5, W_3 = 0.

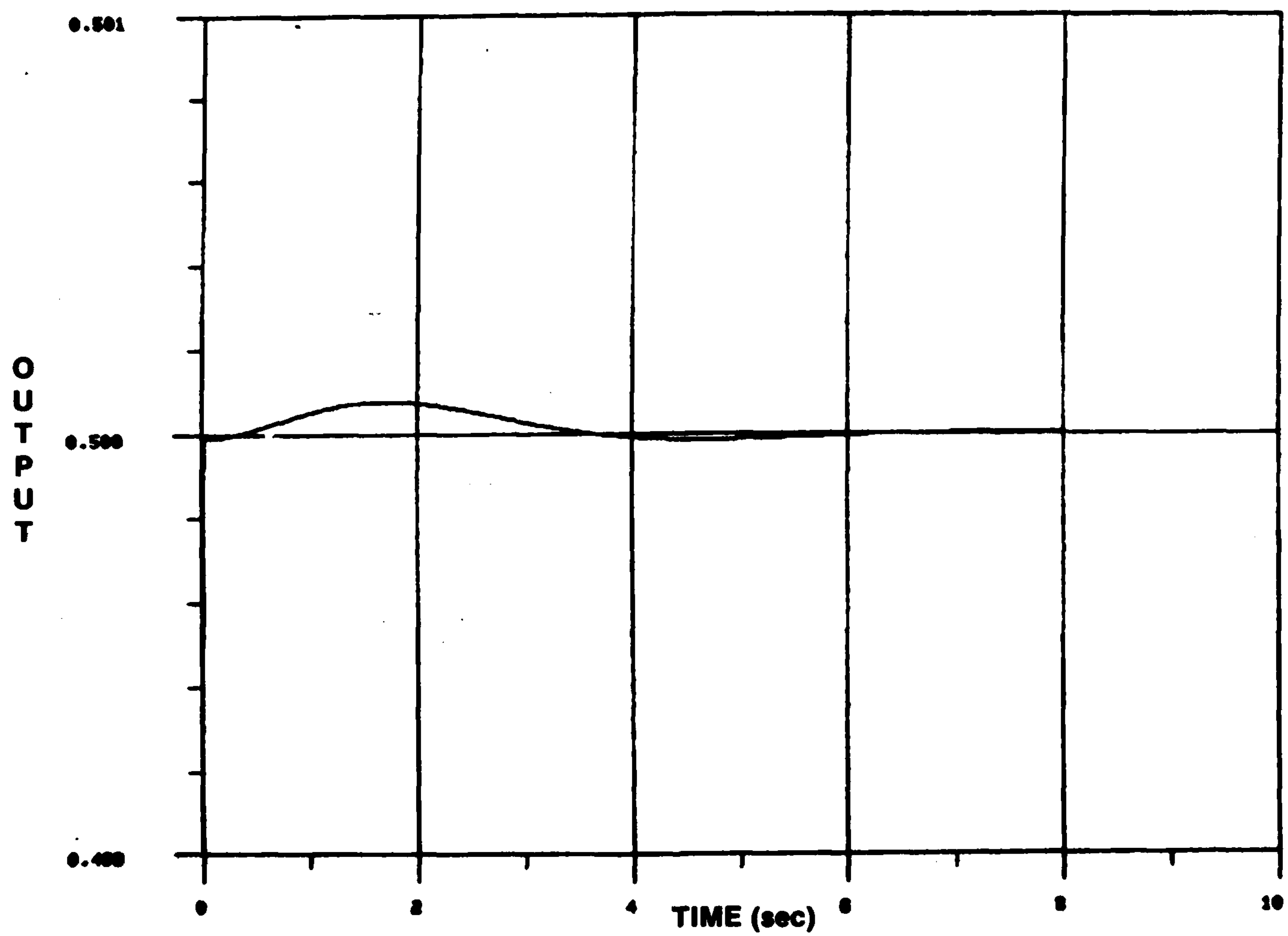

Figure 87. Acceleration loop response, t_f = 66.7 sec, PGO = 0.5, W_3 = 0.5, C_{RAL} = 0.

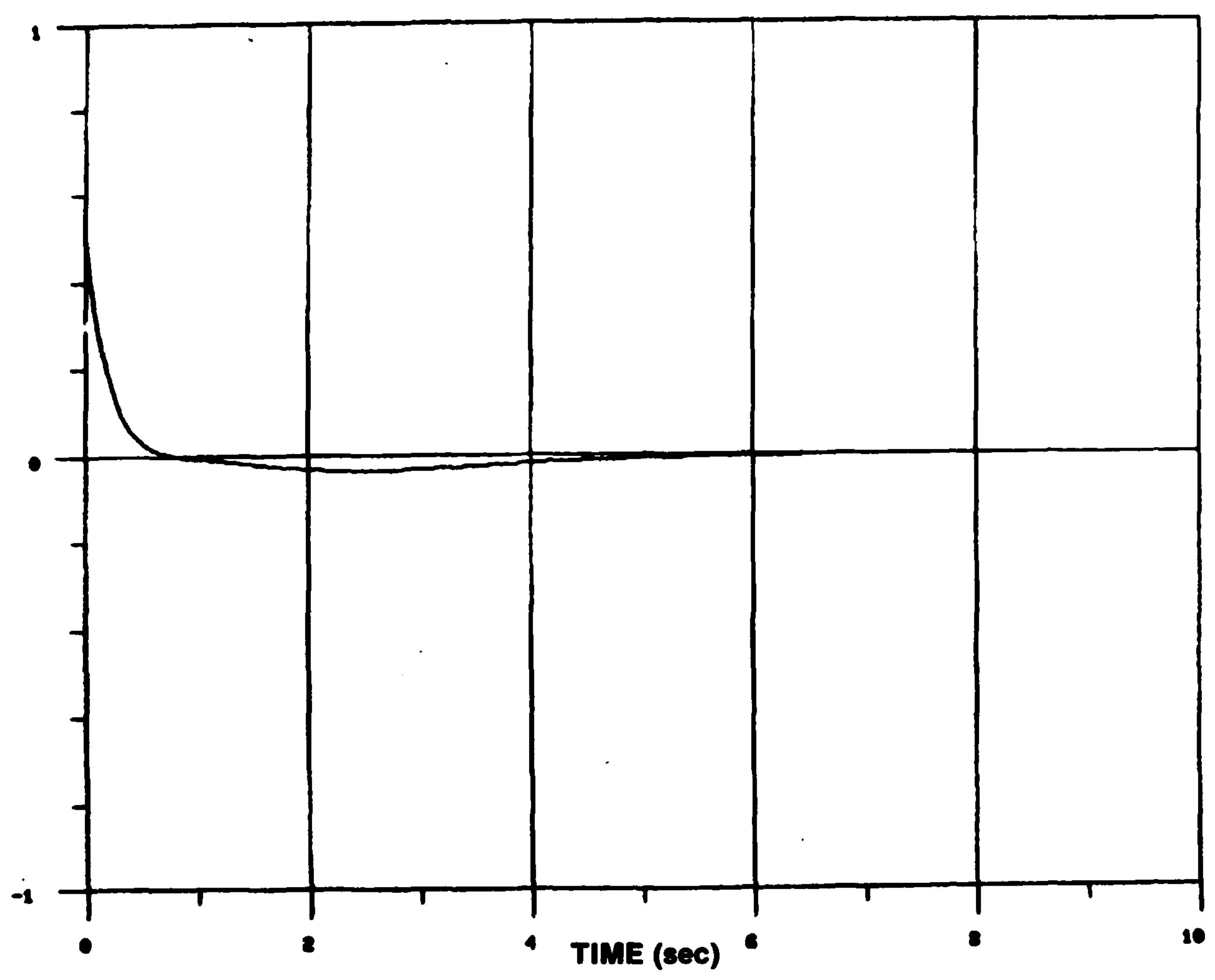

Figure 88. DAC disturbance estimation error, t_f = 66.7 sec, PGO = 0.5, $\hat{W}_3$ = 0.5, C_{RAL} = 0.

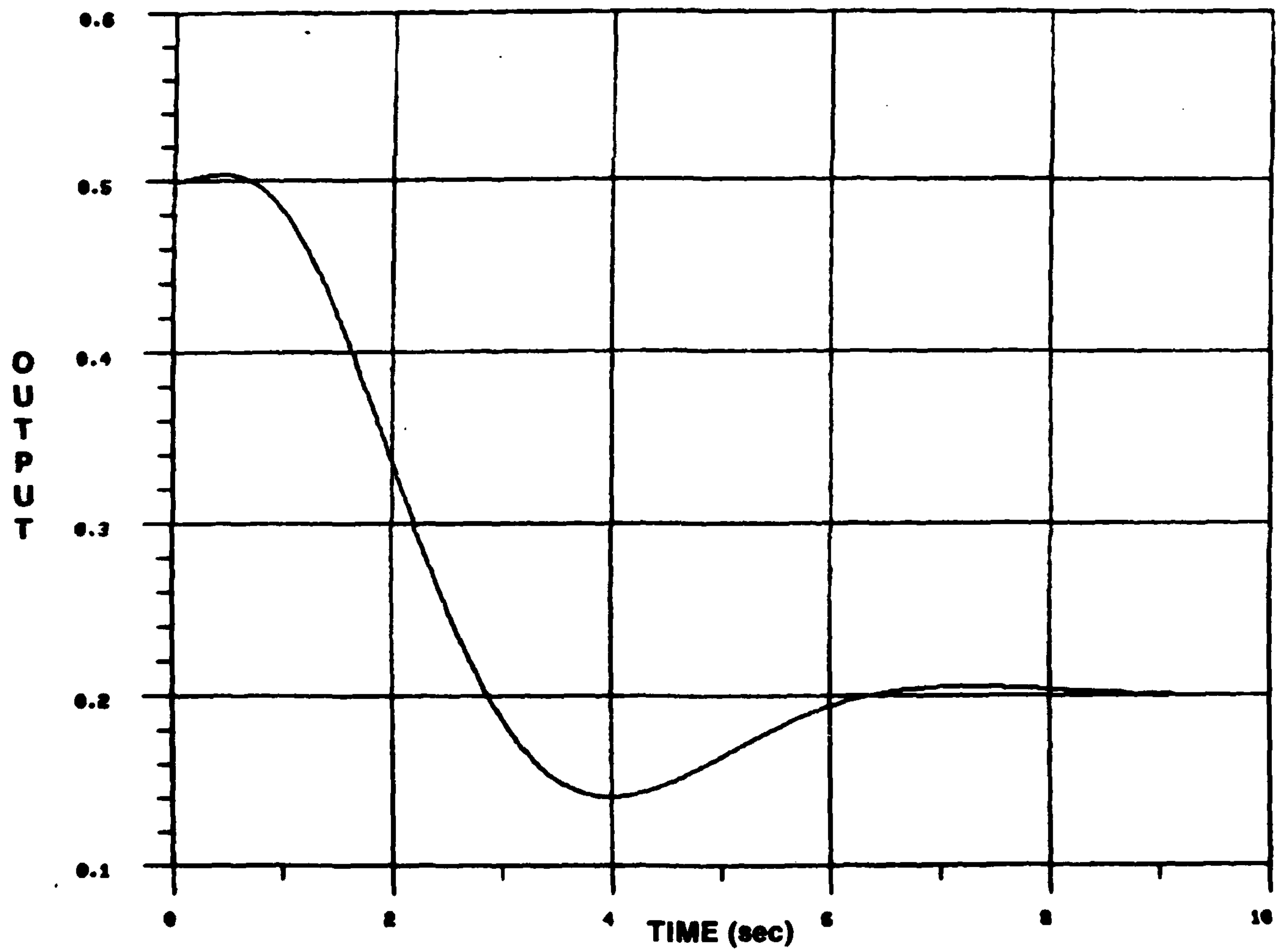

Figure 89. Acceleration loop response, $t_f = 66.7$ sec, PGO = 0.5, $W_3 = 0.5$, $C_{RAL} = -1.50$.

For t_i = 111.4 sec, an input command and disturbance magnitude of 1.0 were again used. *Figures 90* through *94* show the results obtained. Comparing *Figures 90, 92* and *94*, it can be seen that the distrubance effects are removed. The output on *Figure 94* could perhaps be settled out better by more iterations with the state reconstructor roots.

Table 6 gives the components of $\underline{K}_1$ and $\underline{K}_2$ and the roots of Equation (29) used in the above three cases along with the value for C_{RAL} in each case.

TABLE 6. STATE RECONSTRUCTOR DATA AND C_{RAL} FOR ACCELERATION LOOP WITH DISTURBANCE ON OUTPUT

PARAMETER \ TIME POINT (SEC)	9.85	66.7	111.4
k_{11}	3.653	-0.129	-14.54
k_{21}	195.44	-2.033	-184.34
k_{31}	2159.92	-3.68	-605.05
k_{41}	2668.13	-1.776	-483.25
k_{12}	-12.26	-4.584	7.80
k_{22}	0.0	-2.793	-29.47
λ_1	-3.	-1.	-2.
λ_2	-3.	-1.	-2.
λ_3	-4 + j4	-1.5 + j0.25	-3 + j1
λ_4	-4 - j4	-1.5 - j0.25	-3 - j1
λ_5	-6.	-2.	-5.
λ_6	-6.	-2.	-5.
C_{RAL}	-0.12	-1.50	-0.40

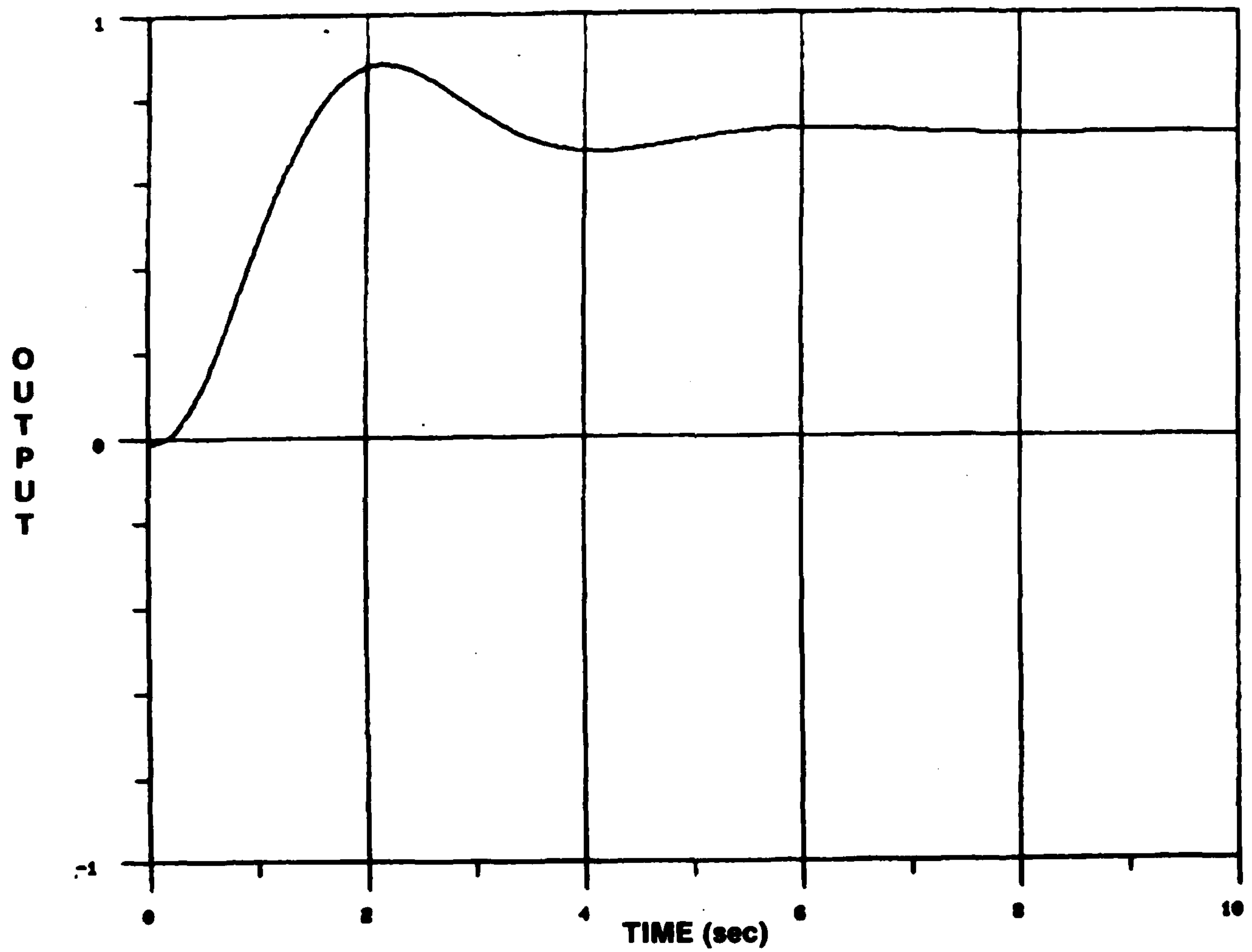

Figure 90. Acceleration loop response, t_f = 111.4 sec, PGO = 1, W_3 = 0.

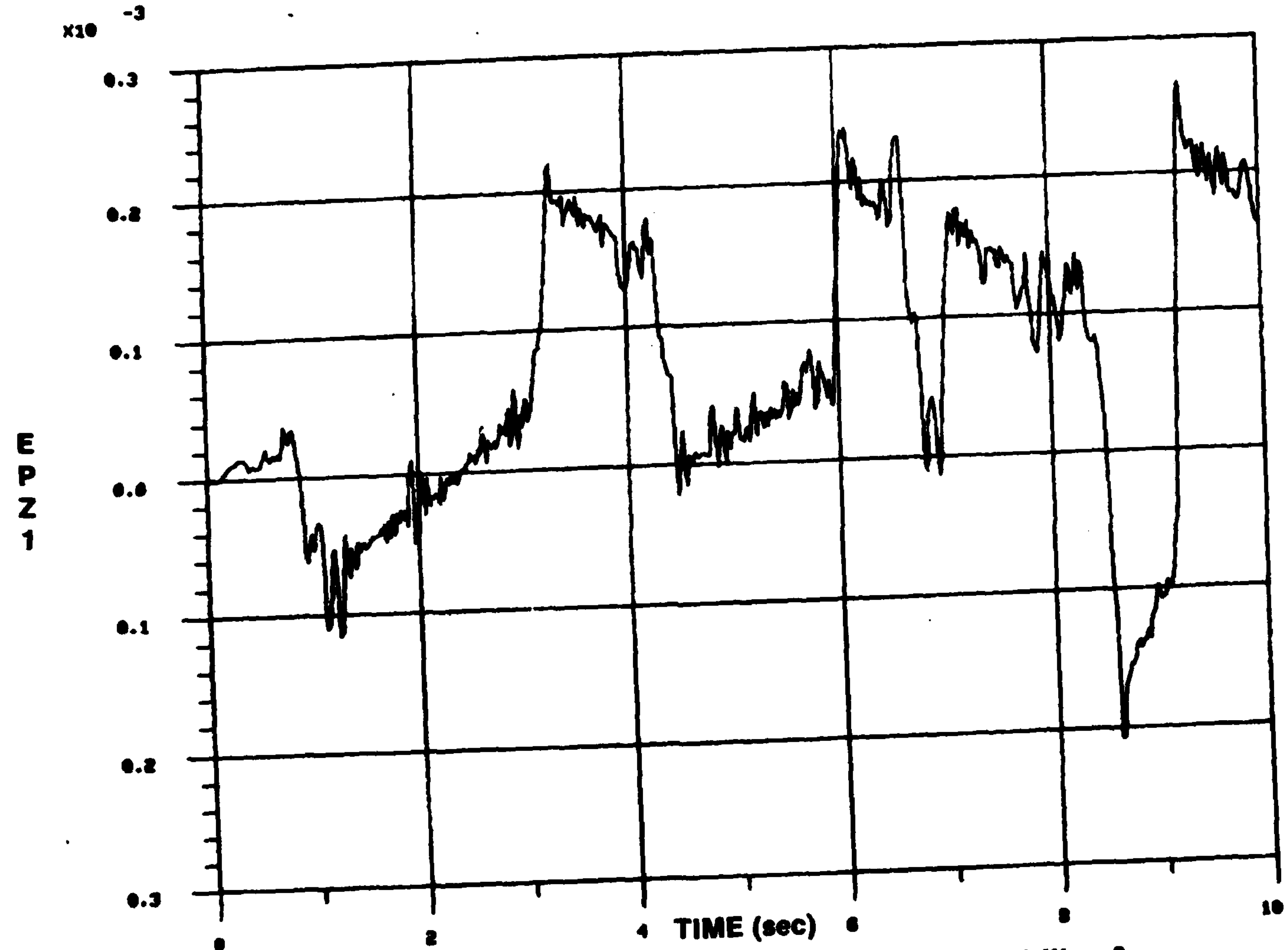

Figure 91. DAC disturbance estimation error, t_f = 111.4 sec, PGO = 1, W_3 = 0.

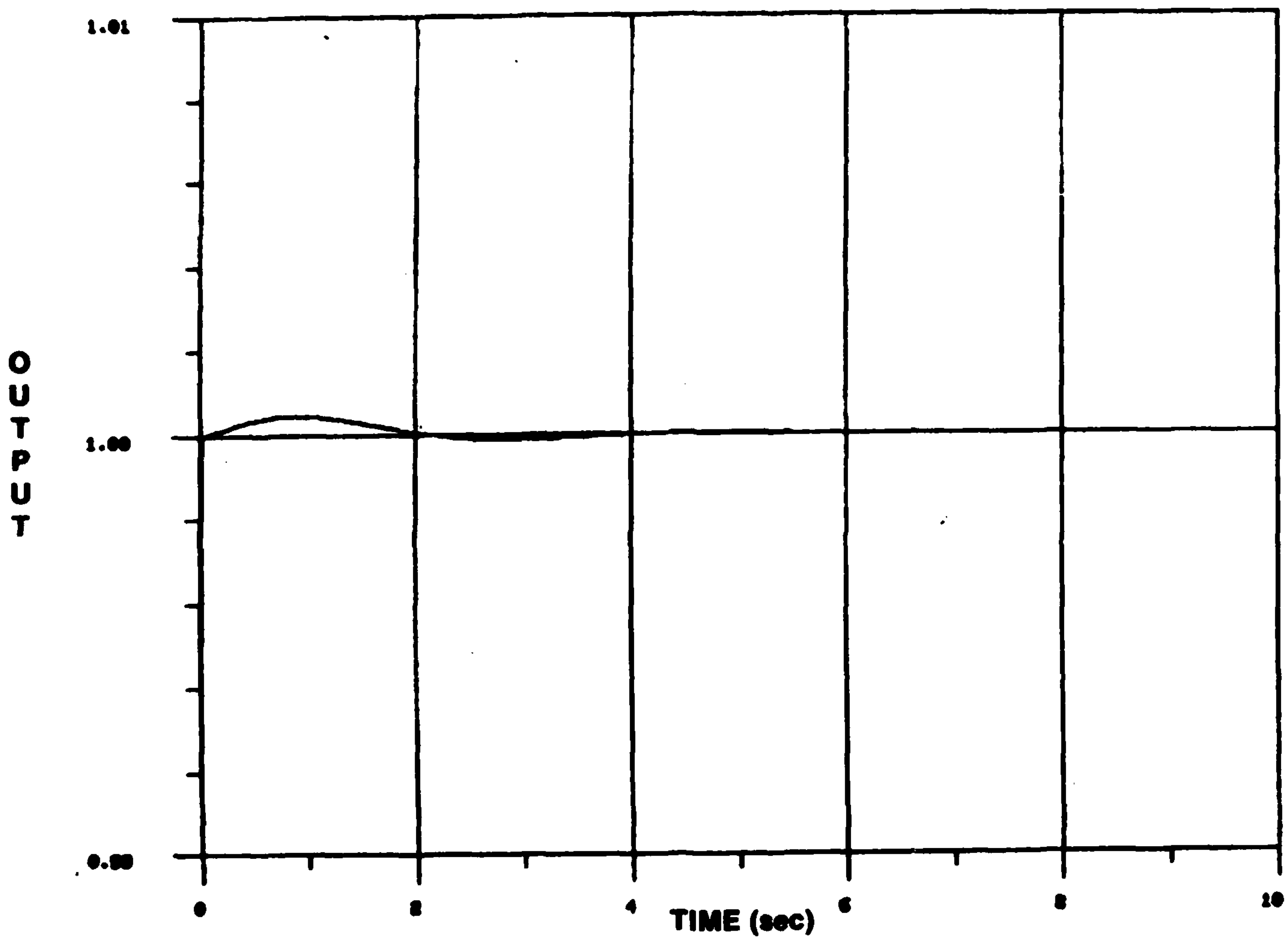

Figure 92. Acceleration loop response, t_f = 111.4 sec, PGO = 1, W_3 = 1, C_{RAL} = 0.

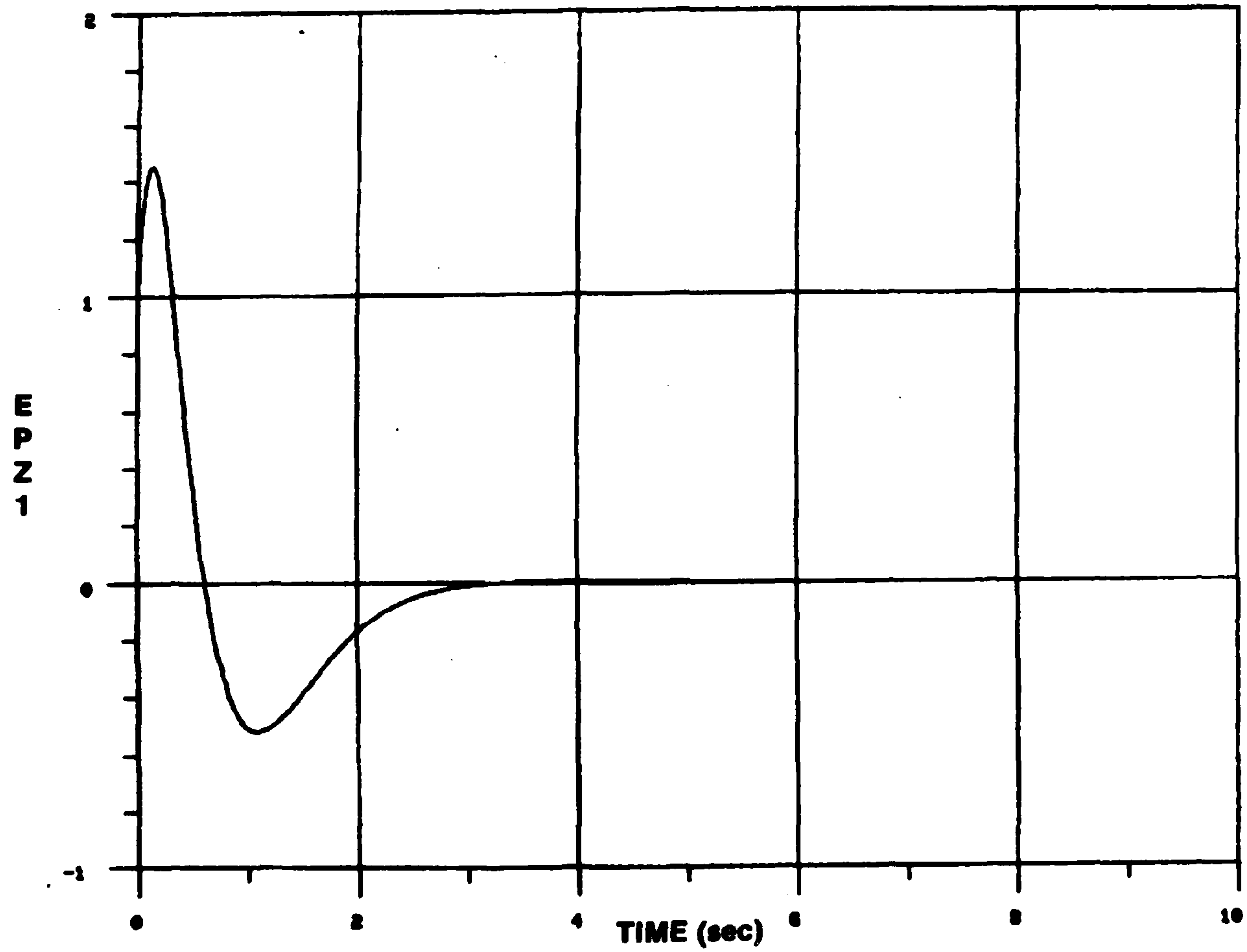

Figure 93. DAC disturbance estimation error, $t_f = 111.4$ sec, PGO = 1, $W_3 = 1$, $C_{RAL} = 0$.

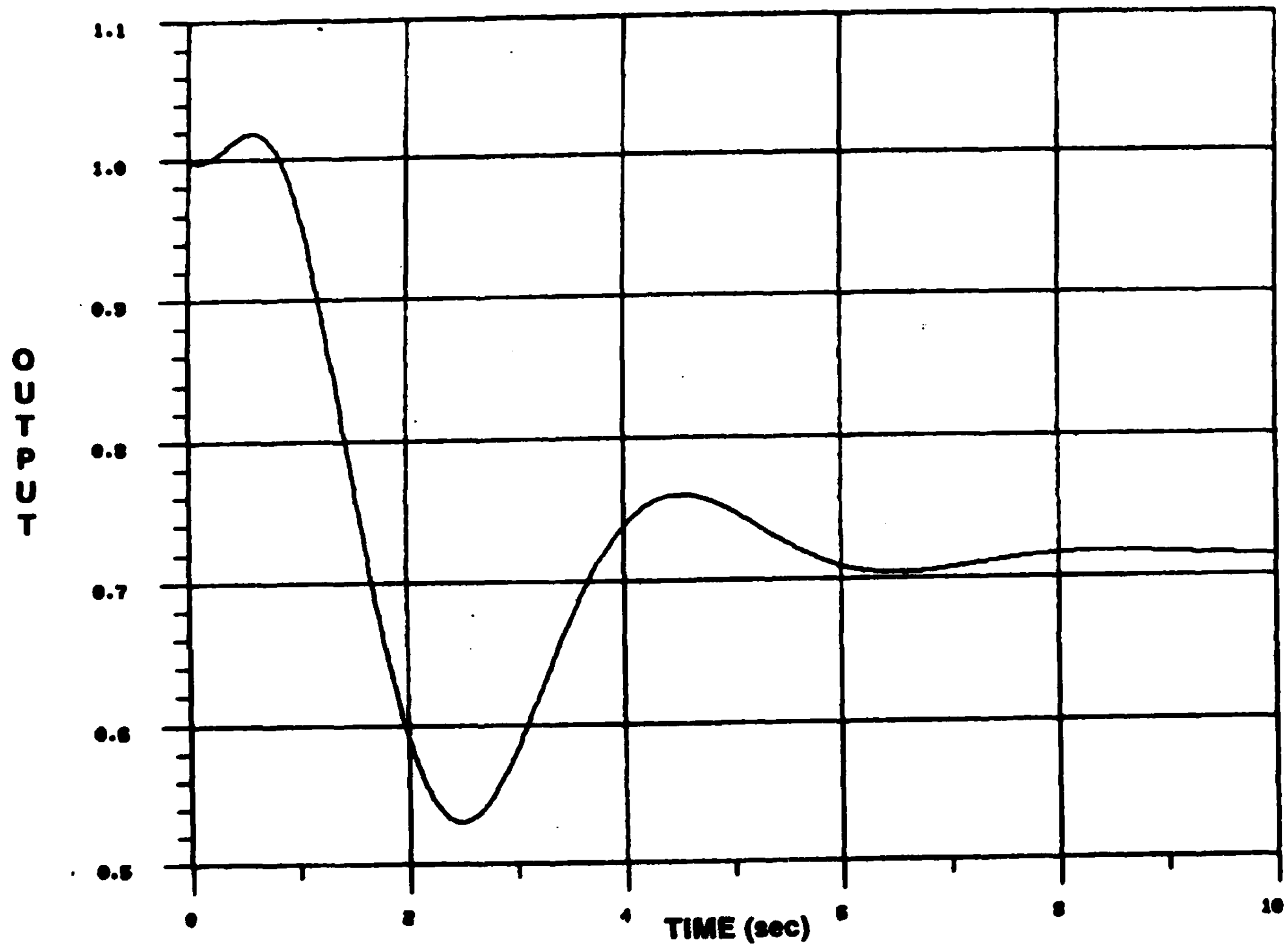

Figure 94. Acceleration loop response, t_f = 111.4 sec, PGO = 1, W_3 = 1, C_{RAL} = -0.4.

C. CONCLUSIONS

From the results, in similar fashion to the rate loop, even though no $\underline{u}_c$ existed which would exactly cancel $\underline{w}_3$, by using a state reconstructor to estimate the value of the disturbance an implementation was possible whereby the effects of $\underline{w}_3$ could be minimized. Again, gain switching would be required to implement this in a system.

8. ACCELERATION LOOP WITH INPUT AND OUTPUT DISTURBANCES

A. MODEL

As a last case for this report, the acceleration loop with $\underline{w}_1$ and $\underline{w}_3$ both included is considered. If the procedures given in previous sections are followed in attempting to derive a DAC for this case, it becomes necessary to evaluate the determinant of an 8×8 matrix to solve for the components of the gain matrices $\underline{K}_1$ and $\underline{K}_2$. This evaluation is tedious at best with many opportunities for mistake.

Therefore, it is of interest to see if the DACs developed in Sections 5 and 7 can be combined in such a fashion as to continue to function as desired in cancelling the effects of $\underline{w}_1$ and $\underline{w}_3$. A block diagram of the proposed combination is shown in *Figure 95*.

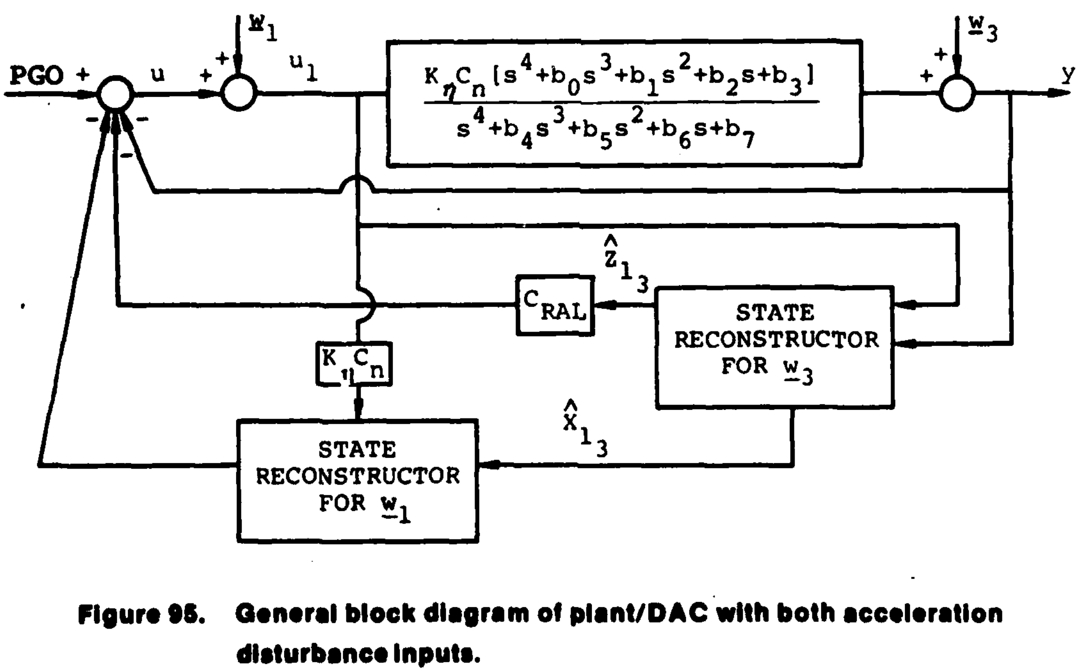

Figure 95. General block diagram of plant/DAC with both acceleration disturbance inputs.

In this development, the same plant state equations as before are used with changes only in nomenclature where necessary. The basic state reconstructor models are as previously developed. In this case, however, the rearrangement of the plant output portion of the data input to the $\underline{w}_1$ state reconstructor should be noted. Since this DAC was developed with no disturbance on the plant output, in order for it to function properly it is necessary to use a plant output with $\underline{w}_3$ removed. This is possible since the $\underline{w}_3$ state reconstructor is also reconstructing the plant states. So, where in Section 5 the plant output is,

$$y = K_\eta C_n u_1 + x_1 ,$$

here it is formulated as

$$y_{PR} = K_\eta C_n u_1 + \hat{x}_{1_3} .$$

Thus, it is important in this application for the $\underline{w}_3$ reconstructor to settle out as rapidly as possible.

B. SIMULATION AND RESULTS

A listing of the simulation is given in Appendix D. In this particular case, only one time point from *Table I* (9.85 sec) was used since the purpose here was to see if two DAC's could be operated successfully in a serially connected mode.

Figures 96 through *98* show the loop output, y, and the disturbance estimation errors, $\epsilon_{\hat{x}_{w1}}$ and $\epsilon_{\hat{x}_{w3}}$, for a nominal run, i.e., input of 1.0, no disturbances. This output compares with similar case results from Section 5 as would be expected. In order to check each reconstructor and see if any undesirable interactions were taking place, two runs were made, one with $w_1 =$ 1., $w_3 = 0.$ and one with $w_1 = 0.$, $w_3 = 1$. The results are shown in *Figures 99* through *104*. In the first case, everything looks okay. In the second case, since the w_3 reconstructor is feeding input to the w_1 reconstructor and has a settling time of several seconds, there are some dynamics induced in the w_1 reconstructor. This in turn causes some dynamics to appear in the output. However, if *Figure 102* is compared to *Figures 96* and *99* on a similar scale, the results do not have such an undesirable appearance. From this it can be seen that some interaction is taking place but not to such an extent that the DAC performance in either case is impaired.

The remainder of the runs were made with disturbence inputs on $\underline{w}_1$ and $\underline{w}_3$ simultaneously. *Figures 105* through *107* give results for $w_1 = 1.$, $w_3 = 2.$; *Figures 108* through

110 for $w_1 = 1. + 0.2t$, $w_3 = 0.5 + 0.1t$ and *Figures 111* through *113* for $w_1 = 1.-0.2t$, $w_3 = 0.5 + 0.5t$. In all these cases, even though the induced dynamics are noted in $\epsilon_{z_{w1}}$, the DAC's performed their function of cancelling the disturbance effects.

C. CONCLUSIONS

From the results in this section it would appear that it is possible to design DAC's for separate disturbances in different parts of a loop, thereby simplifying the size of the matrices involved in the calculations, and combine them in a simple manner to achieve the desired results.

9. CONCLUSIONS

Conclusions have been presented in Sections 5 through 8 regarding the results obtained with the design in each section. Overall, it has been shown that it was possible to cancel out or minimize the effects of the disturbances modeled herein by use of DAC techniques. It was also shown that it was possible to combine two separate DAC's, designed for disturbances at different places in the plant, into a functioning unit which would still perform its overall purpose. This is especially important since the size of the matrices involved in designing a "full-dimensional" observer is directly related to the dimensions of the plant plus disturbance models. Thus, any procedure which can reduce the dimensionality involved is important. In this regard, several of the designs here might be redone utilizing a "reduced-order" observer to see how well such a DAC would perform.

Although it would appear from the results obtained here that a DAC might be very useful in cancelling out unwanted disturbances, the only way to really be sure how one would function in a system application would be to implement one in a 6-DOF simulation and fly it with a severe program of varying disturbance vectors.

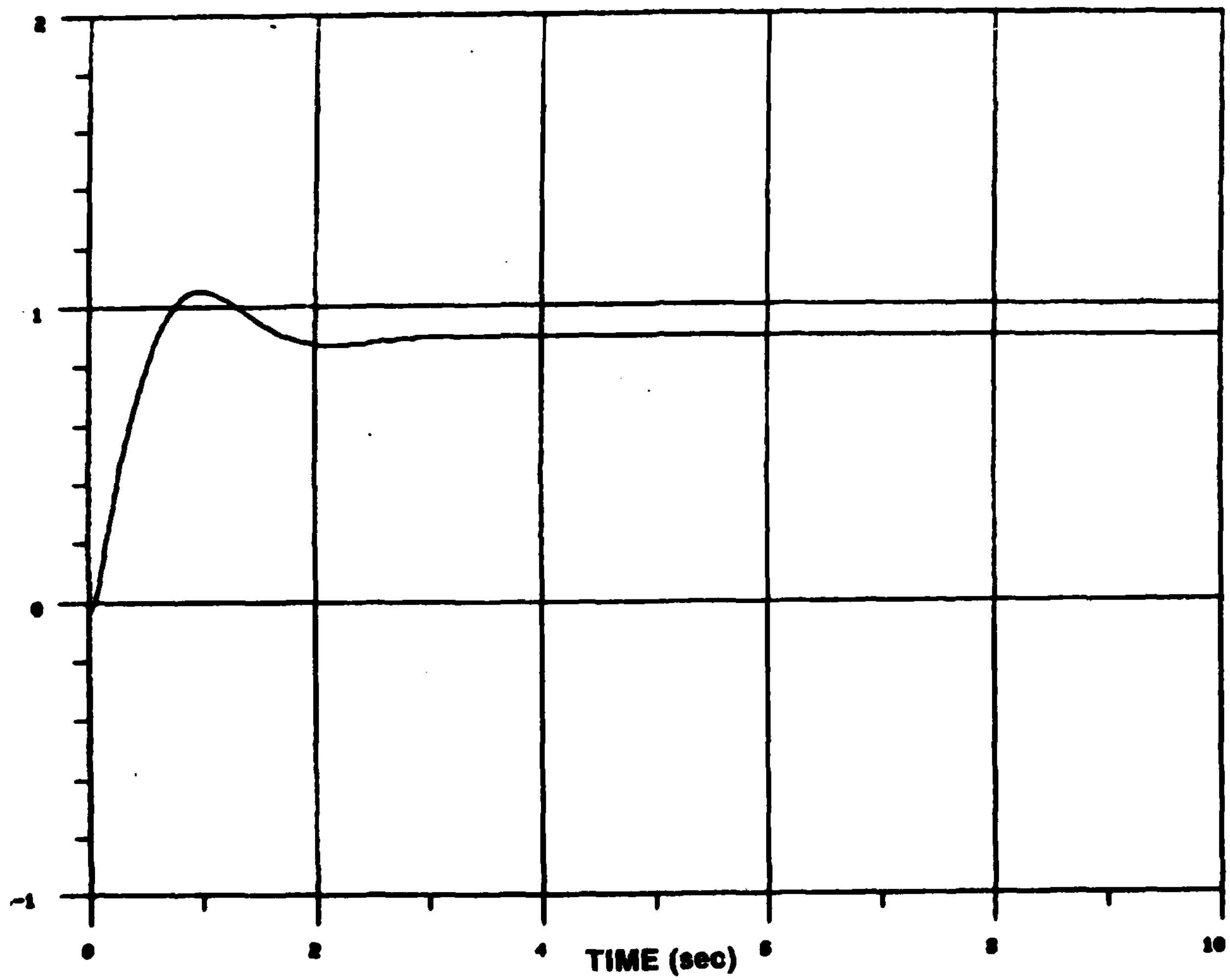

Figure 96. Acceleration loop response, t_f = 9.85 sec, PGO = 1, $W_1 = W_3 = 0$.

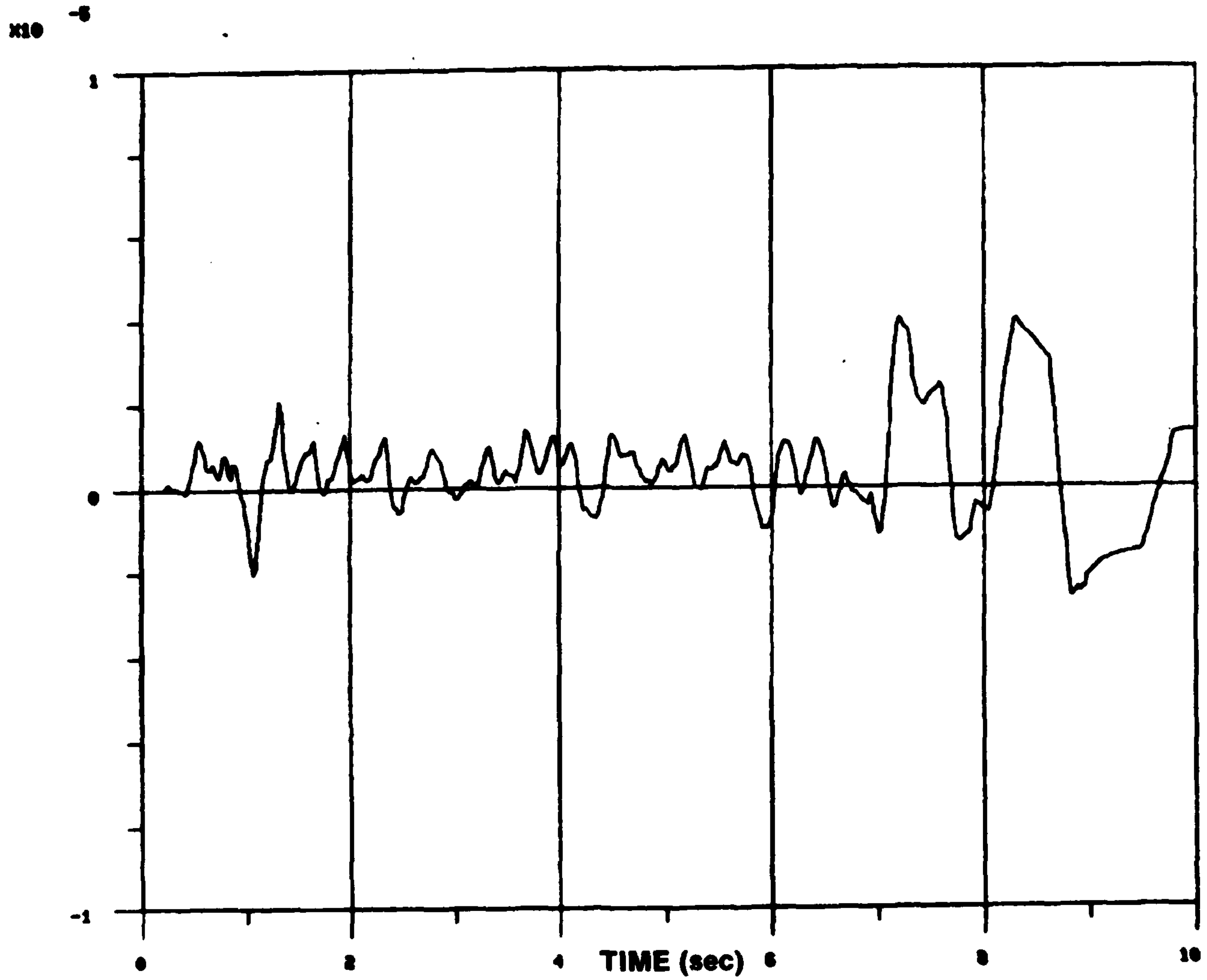

Figure 97. DAC disturbance estimation error for W_1, t_f = 9.85 sec, PGO = 1, $W_1 = W_3 = 0$.

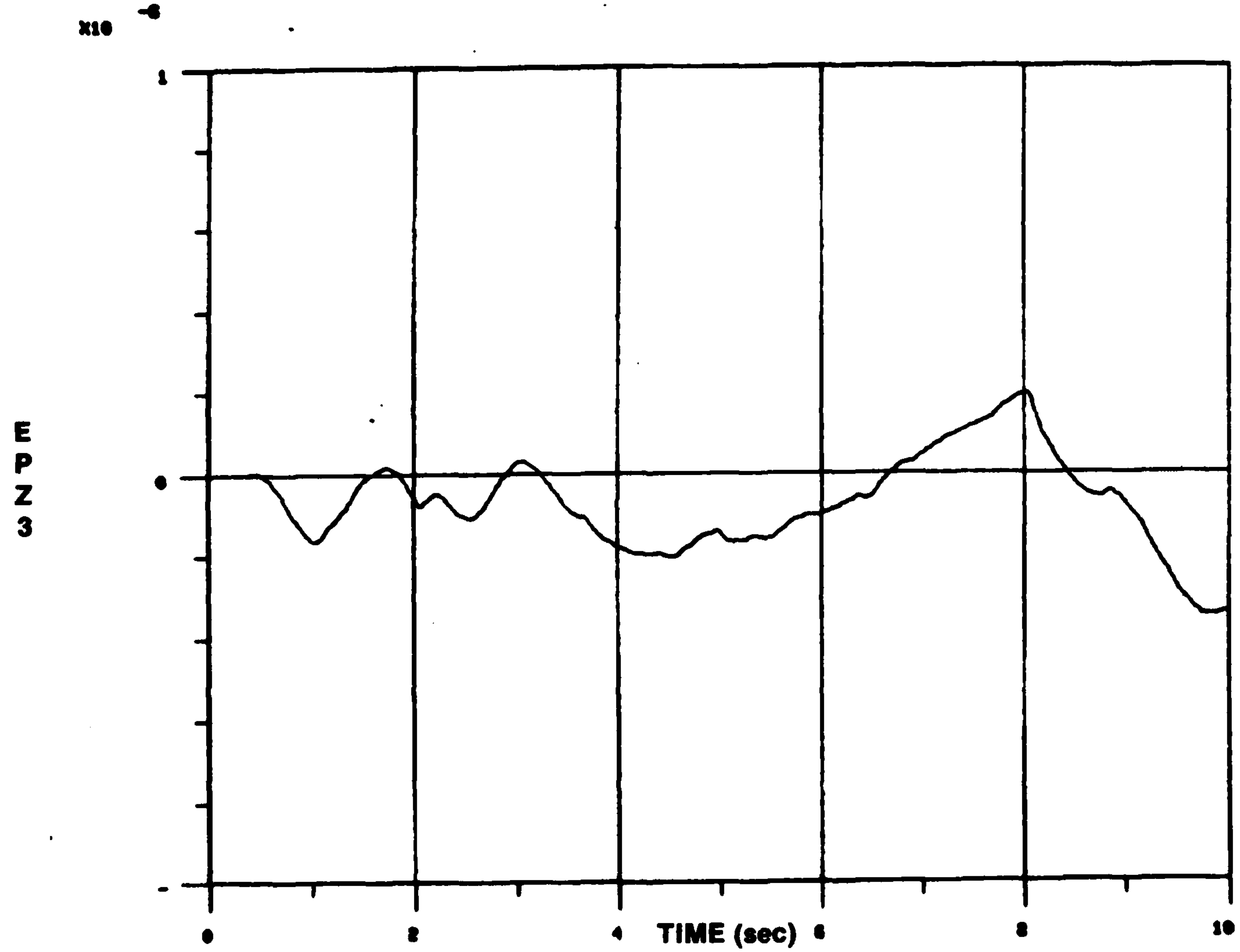

Figure 98. DAC disturbance estimation error for W_3, $t_f = 9.85$ sec, PGO = 1, $W_1 = W_3 = 0$.

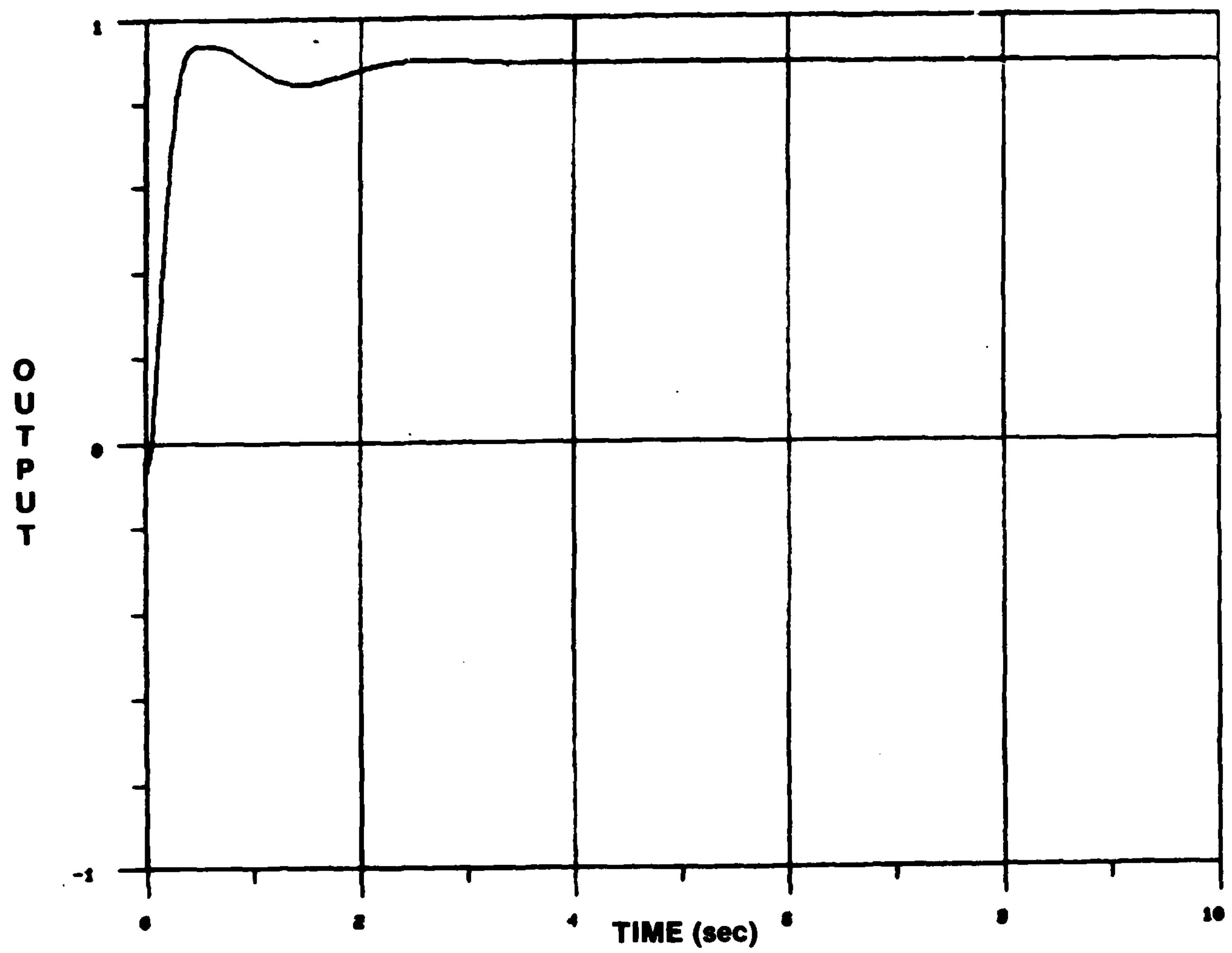

Figure 95. Acceleration loop response $t_f = 9.85$ sec, PGO = 1, $W_1 = 1$, $W_3 = 0$.

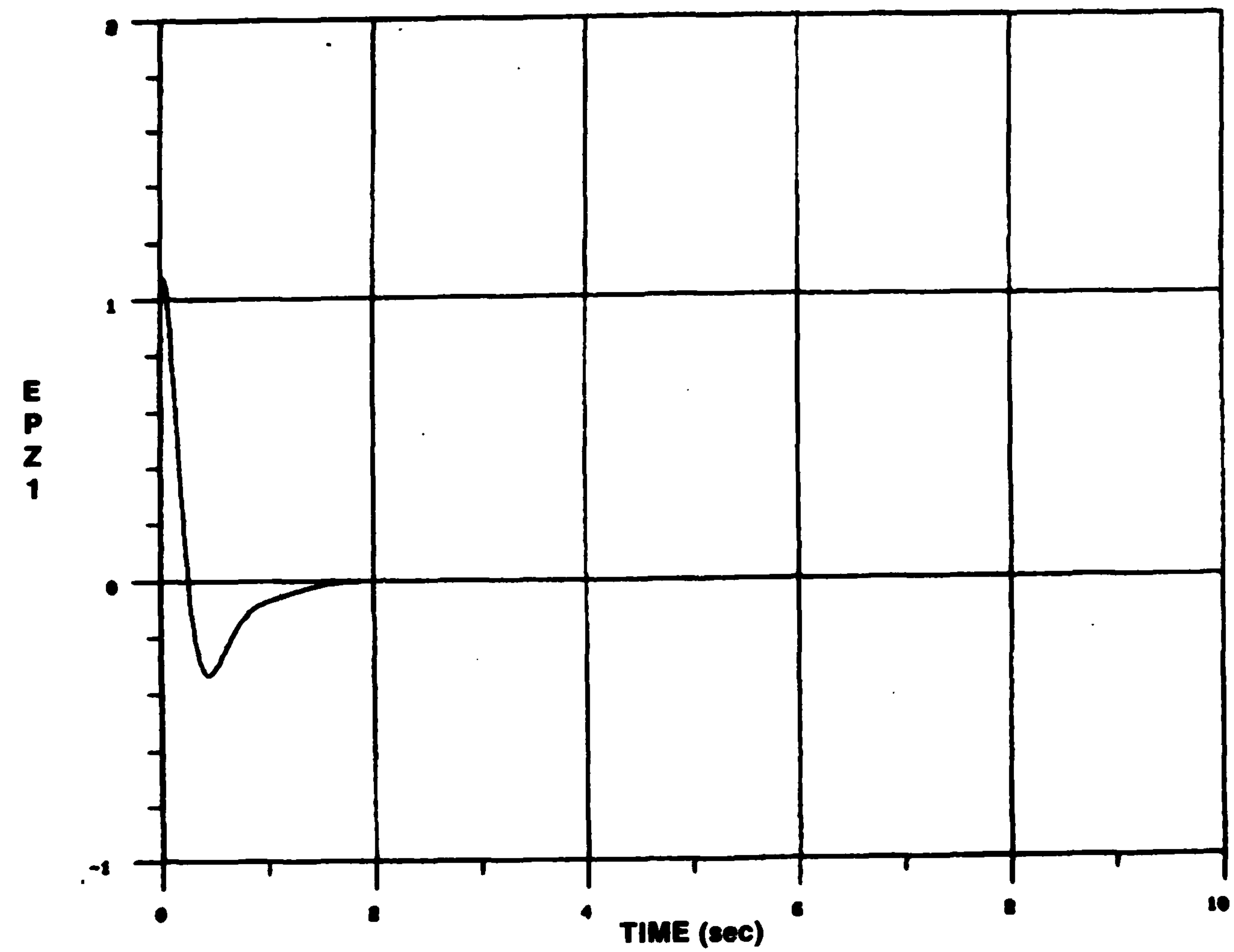

Figure 100. DAC disturbance estimation error for W_1, t_f = 9.85 sec, PGO = 1, W_1 = 1, W_3 = 0.

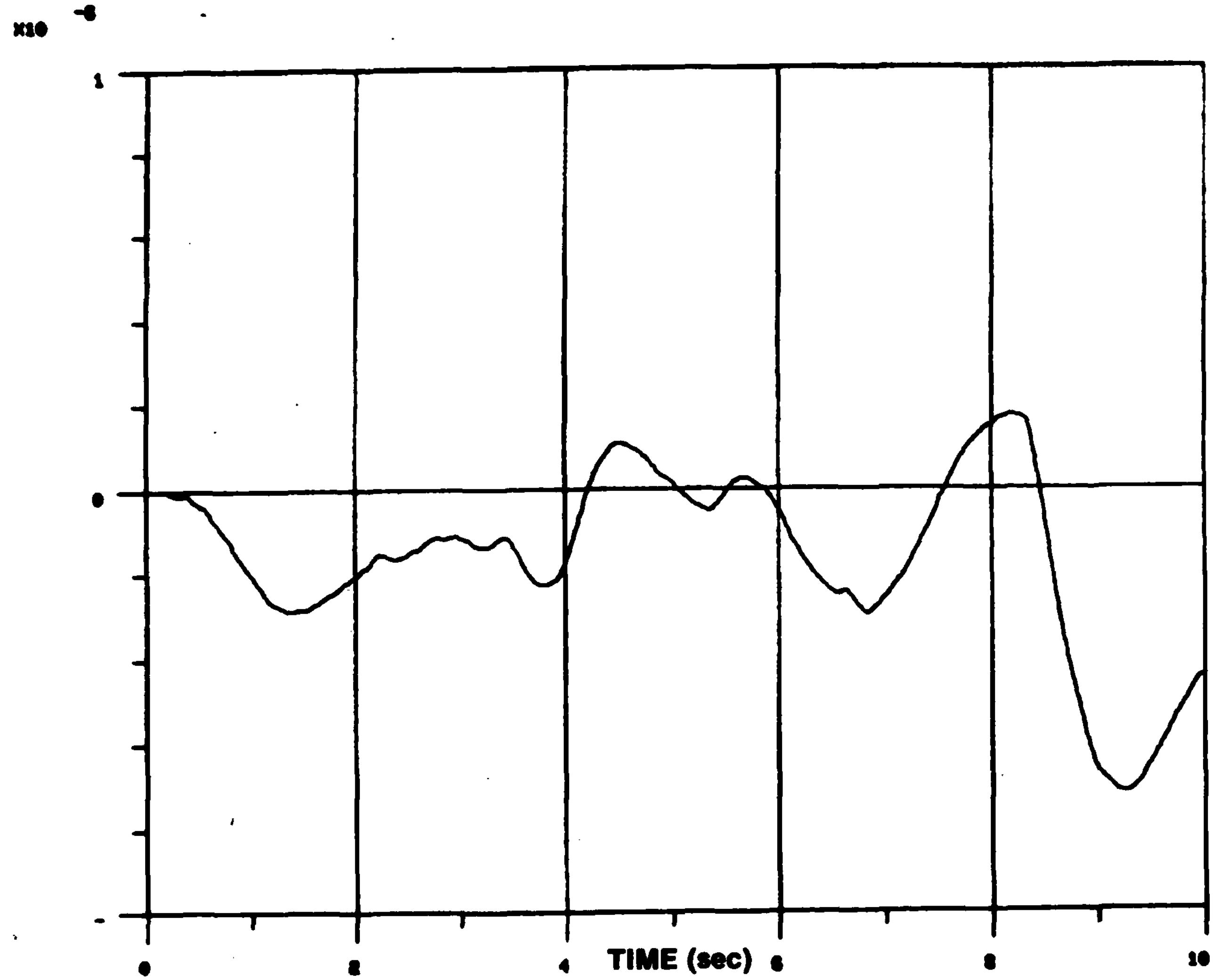

Figure 101. DAC disturbance estimation error for W_3, t_f = 9.85 sec, PGO = 1.0, W_1 = 1.0, W_3 = 0.

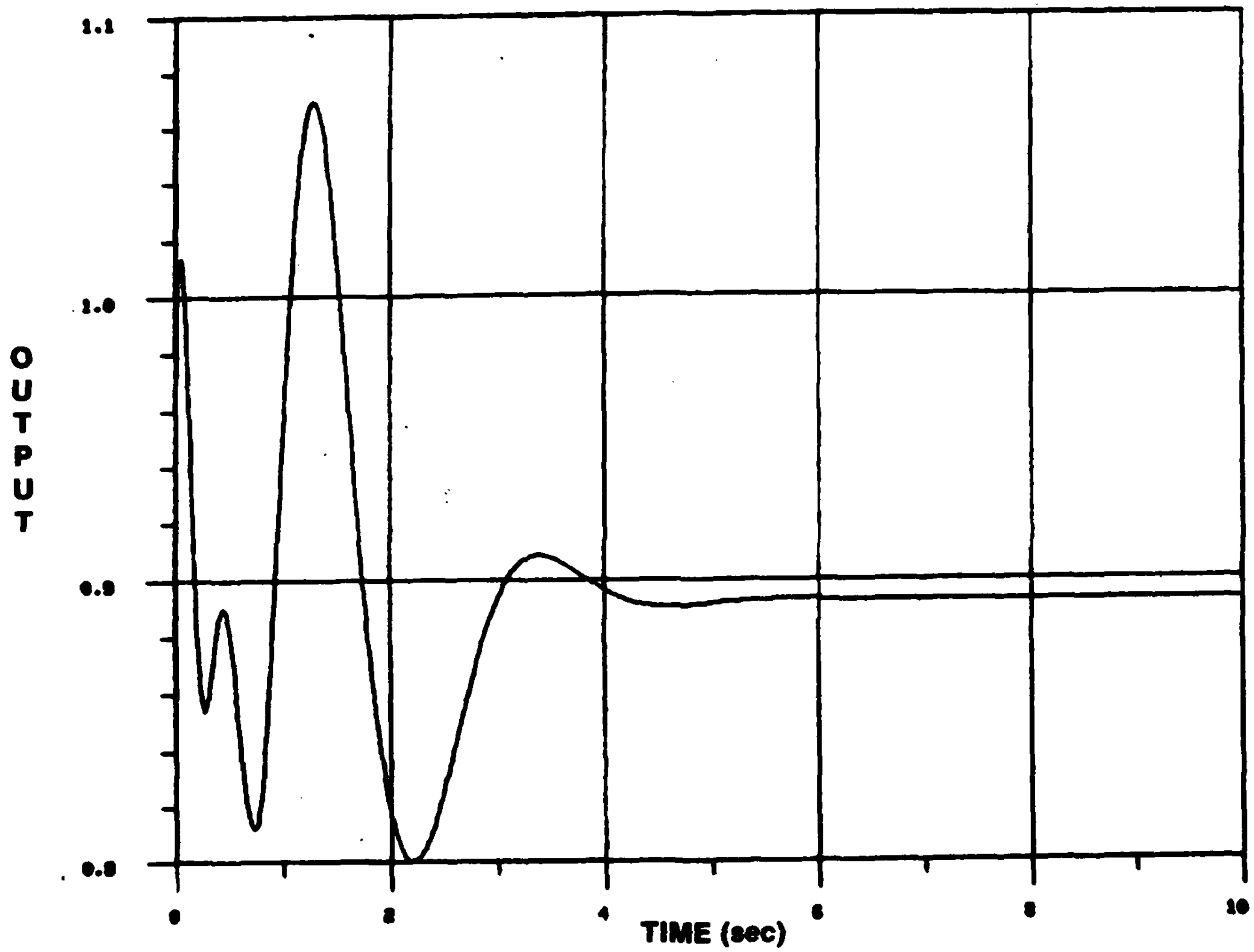

Figure 102. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_1 = 0, W_3 = 1.

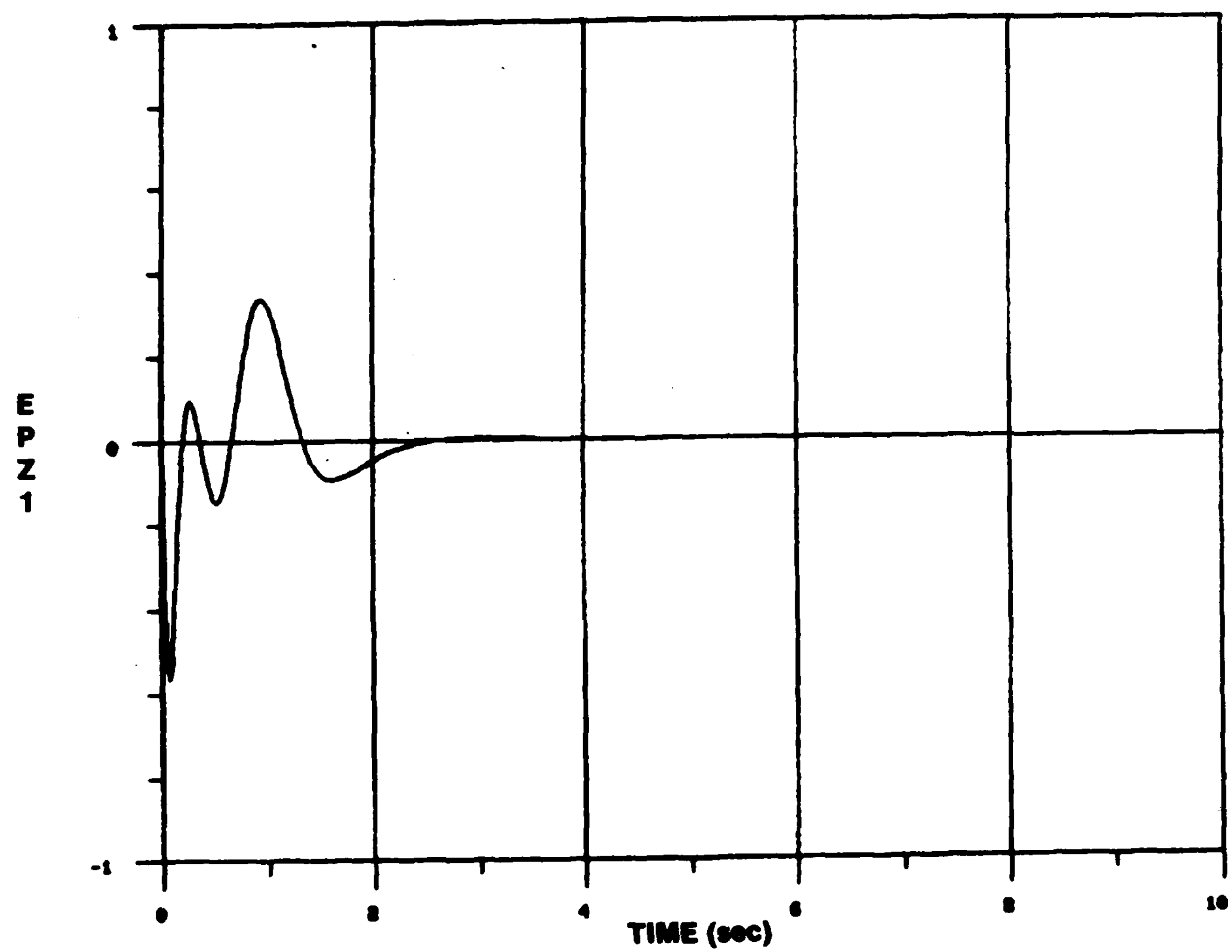

Figure 103. DAC disturbance estimation error for W_1, t_f = 9.85 sec, PGO = 1, W_1 = 0, W_3 = 1.

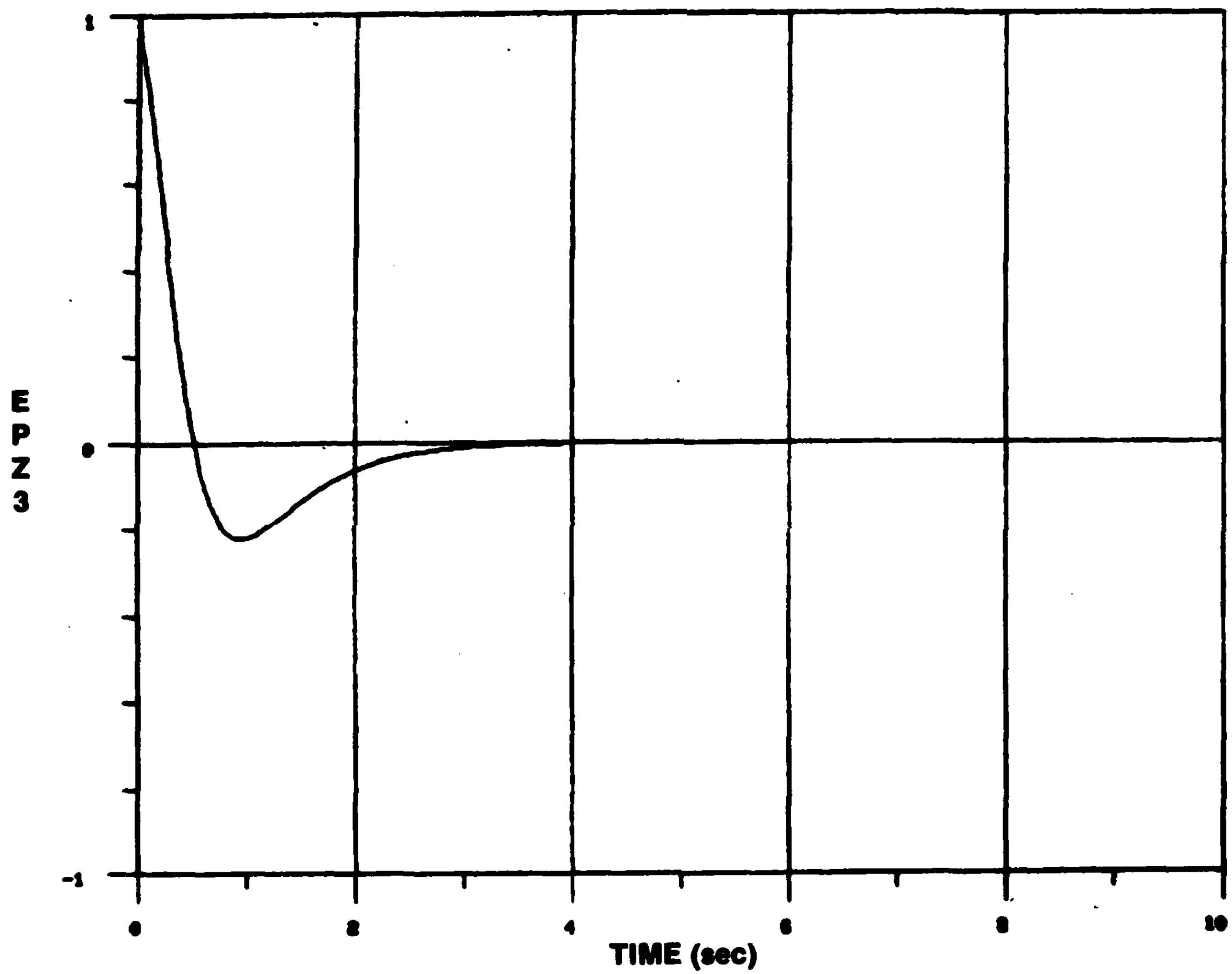

Figure 104. DAC disturbance estimation error for W_3, t_f = 9.85 sec, PGO = 1, W_1 = 0, W_3 = 1.

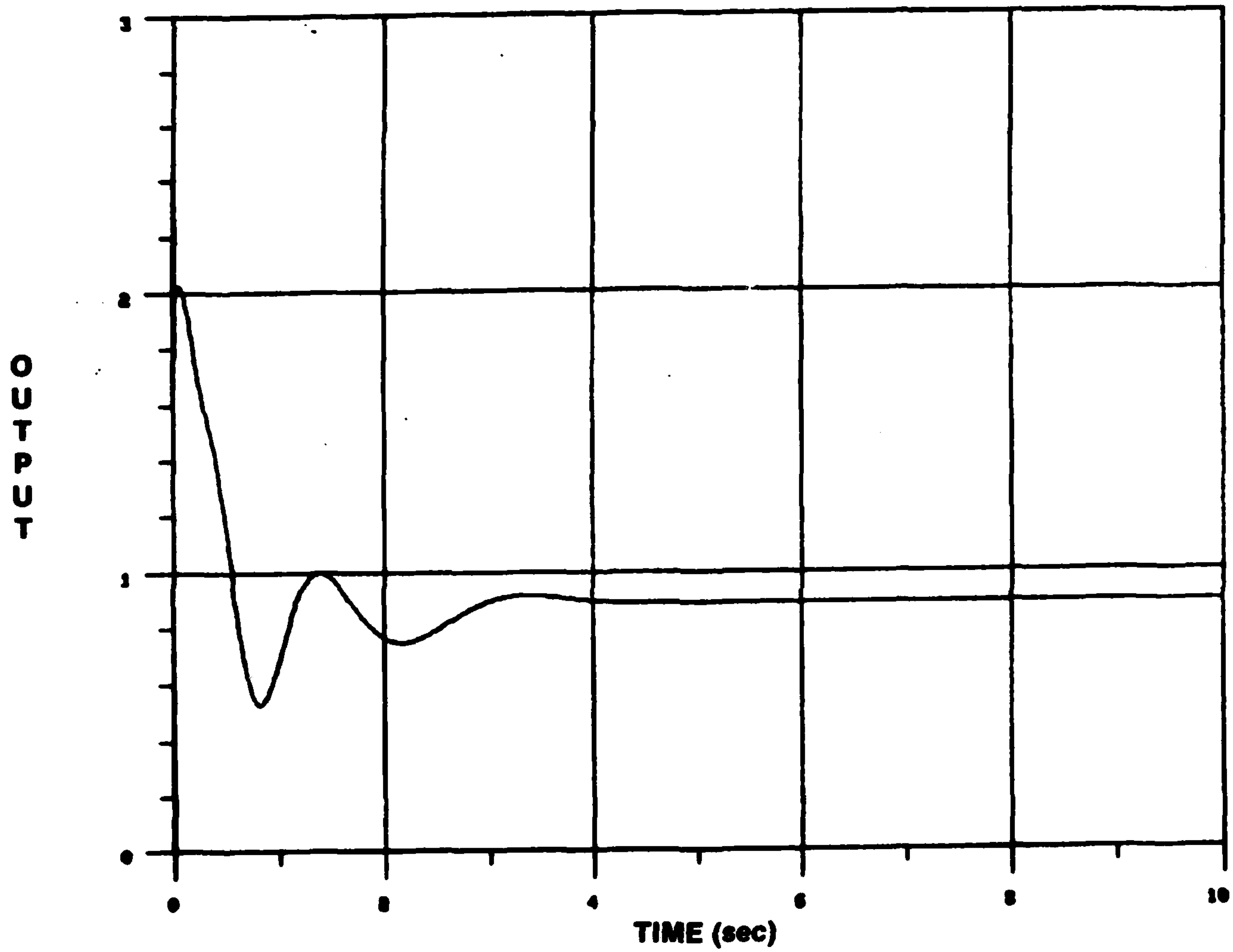

Figure 105. Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_1 = 1, W_3 = 2.

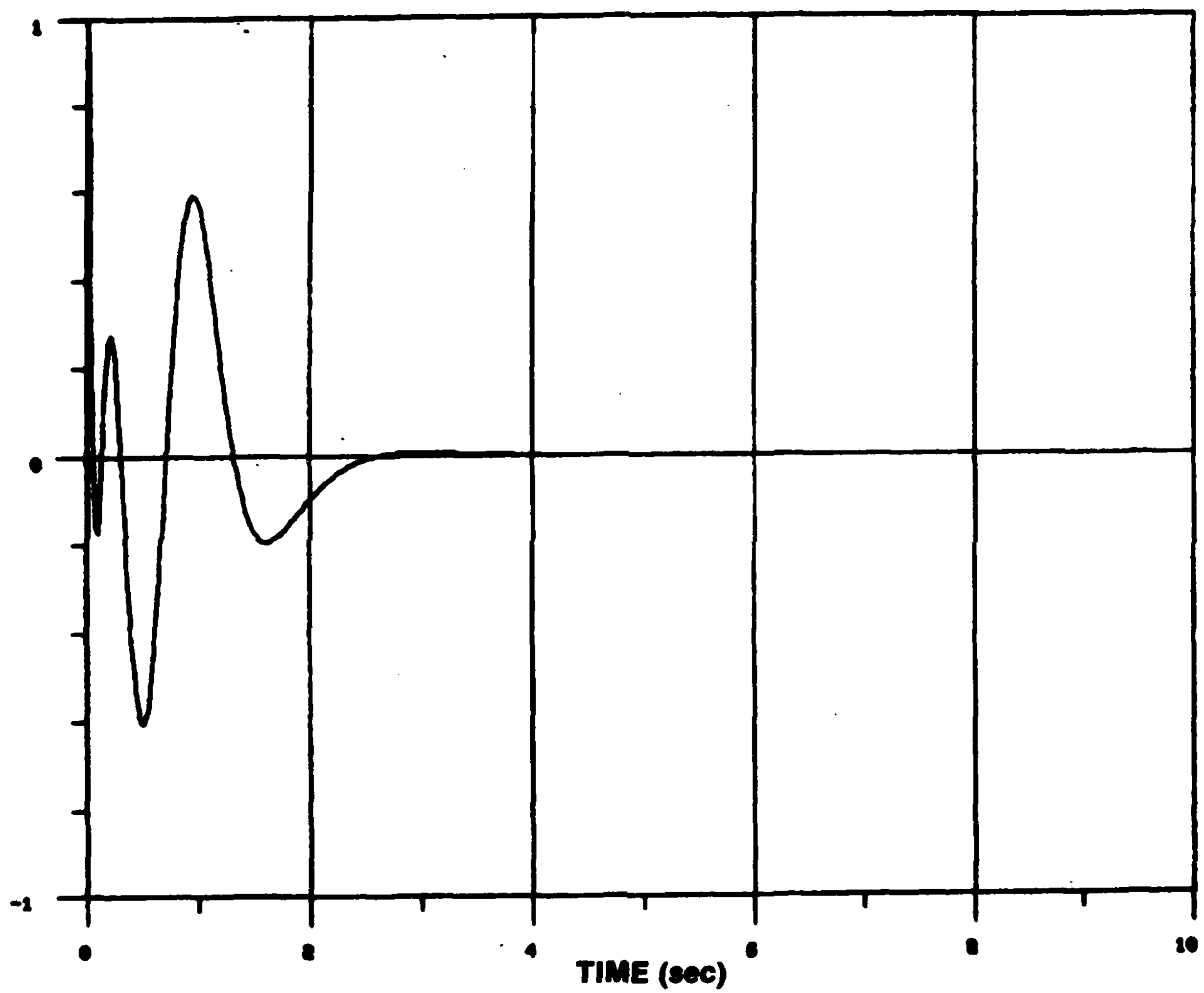

Figure 106. DAC disturbance estimation error for W_1, t_f = 9.85 sec, PGO = 1, W_1 = 1, W_3 = 2.

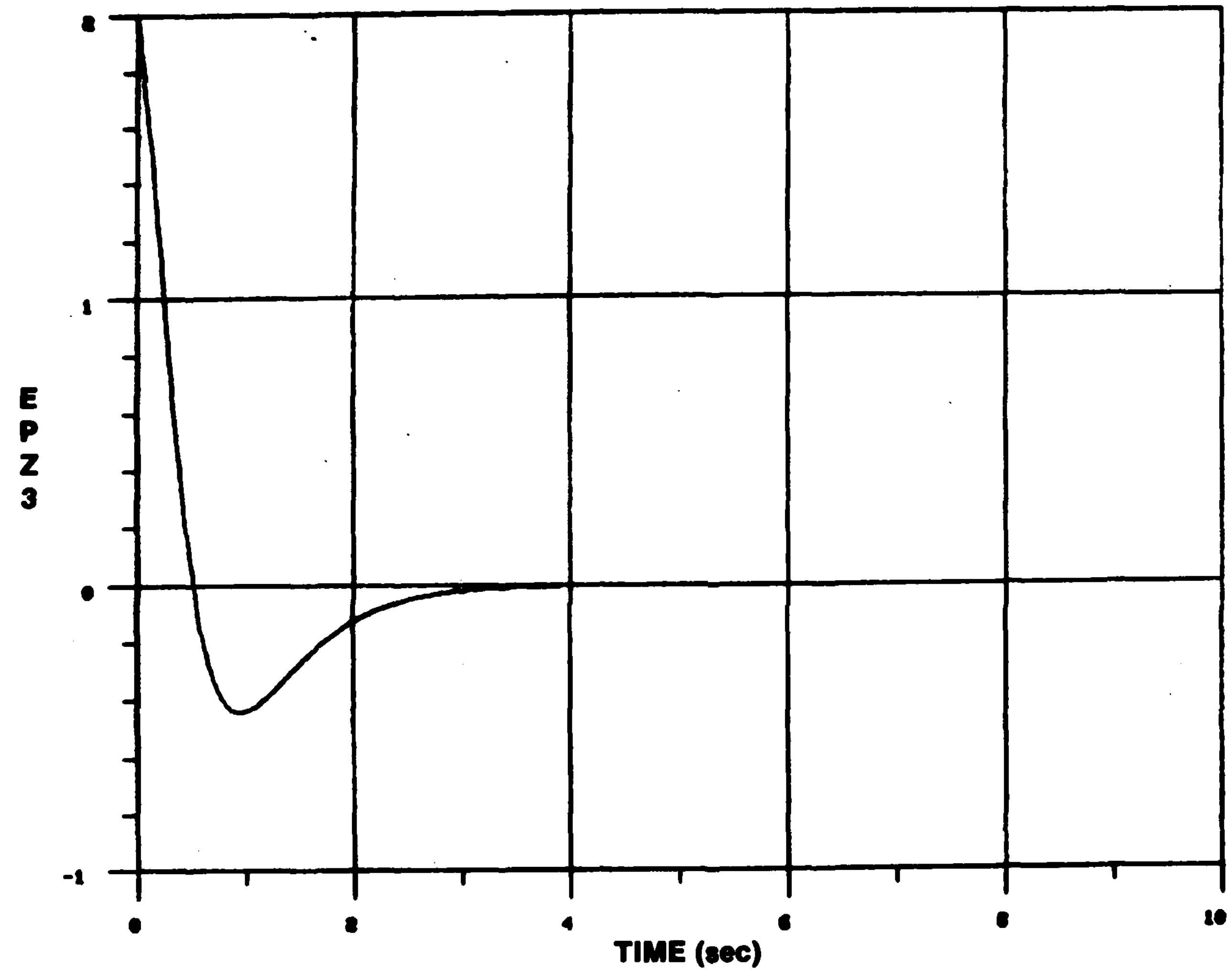

Figure 107. DAC disturbance estimation error for W_3, $t_f = 9.85$ sec, PGO = 1, $W_1 = 1$, $W_3 = 2$.

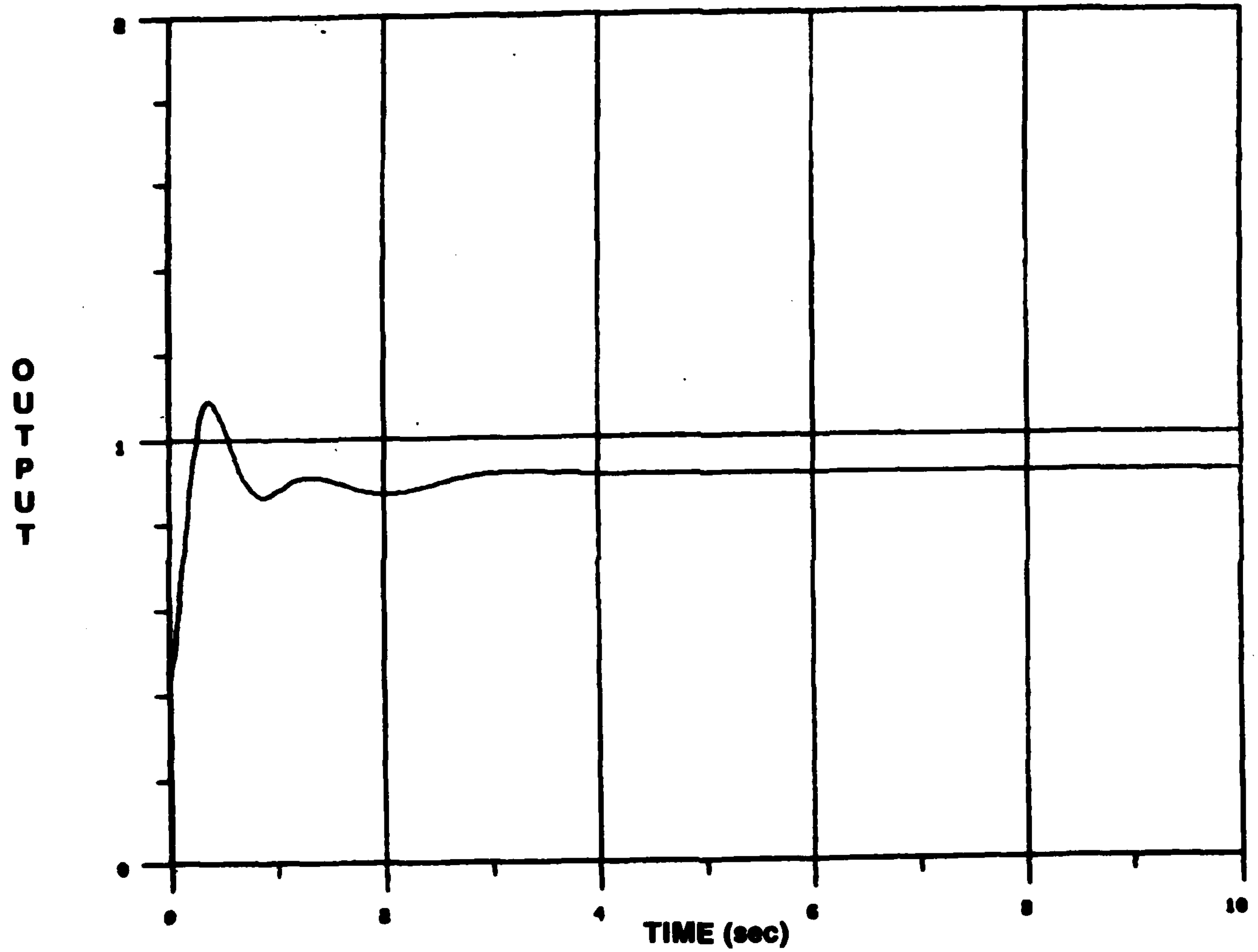

Figure 108. Acceleration loop response, $t_f = 9.85$ sec, PGO = 1, $W_1 = 1 + 0.2t$, $W_3 = 0.5 + 0.1t$.

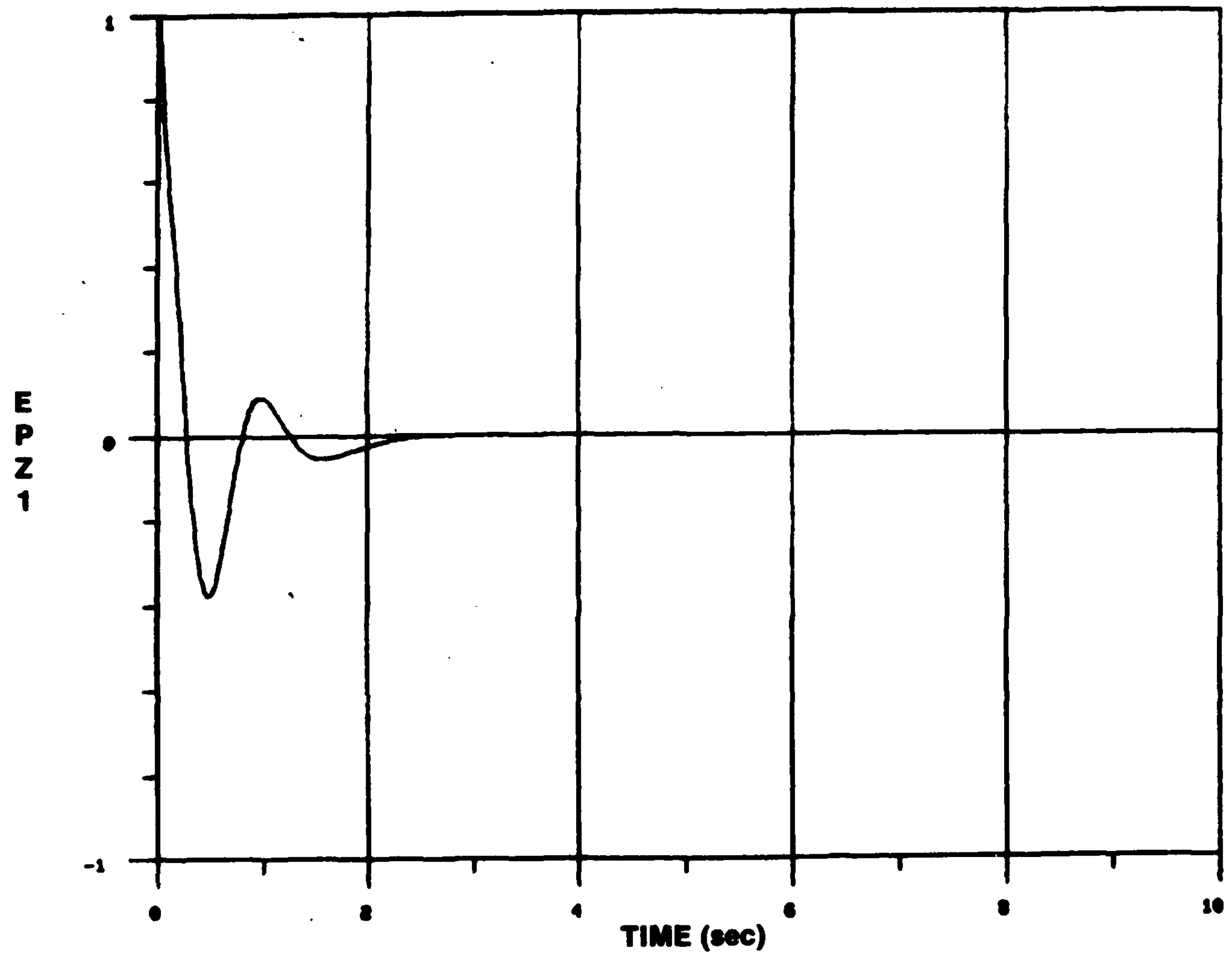

Figure 109. DAC disturbance estimation error for W_1, $t_f = 9.85$ sec, PGO = 1, $W_1 = 1 + 0.2t$, $W_3 = 0.5 + 0.1t$.

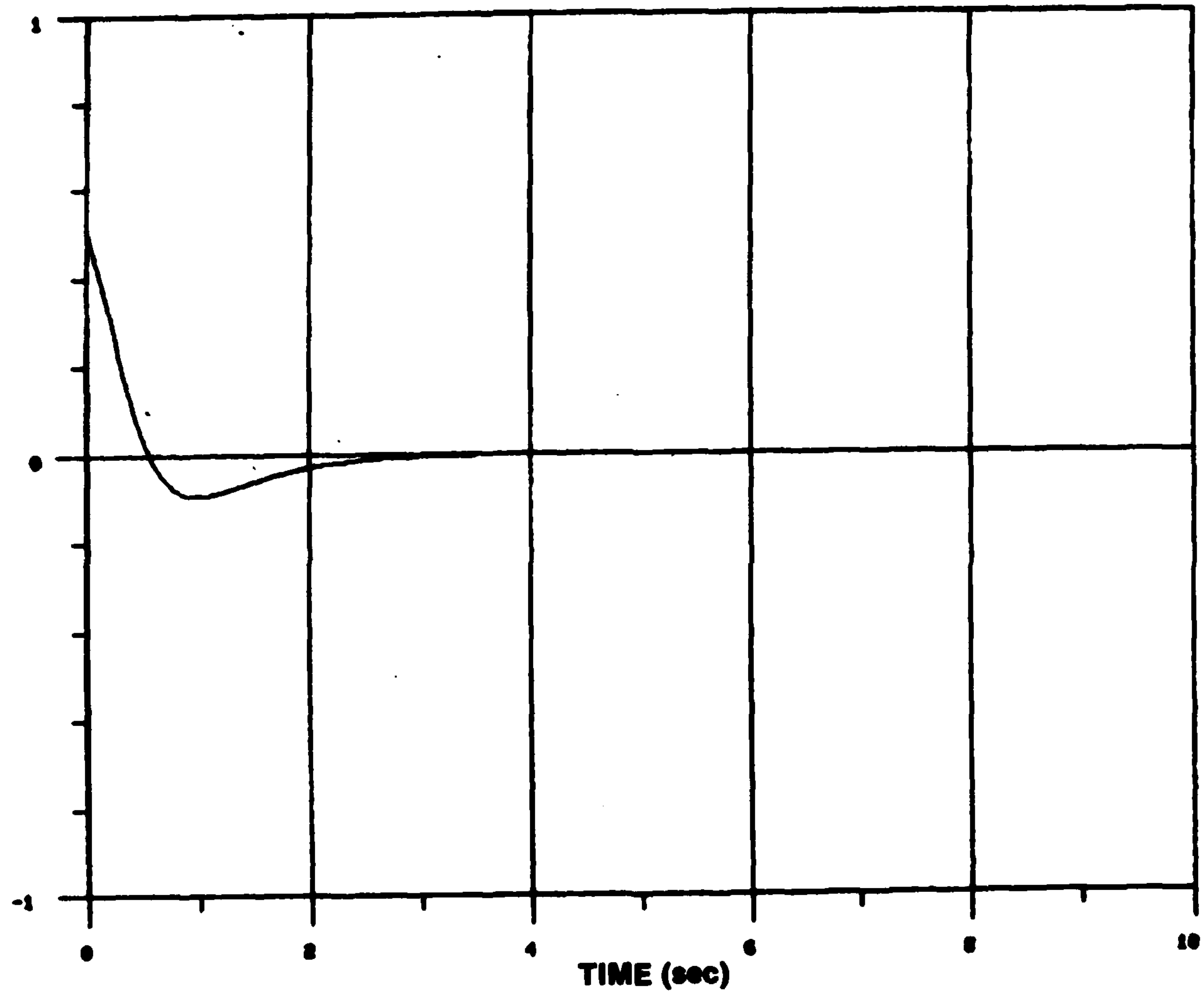

Figure 110. DAC disturbance estimation error in W_3, $t_f = 9.85$ sec, PGO = 1, $W_1 = 1 + 0.2t$, $W_3 = 0.5 + 0.1t$.

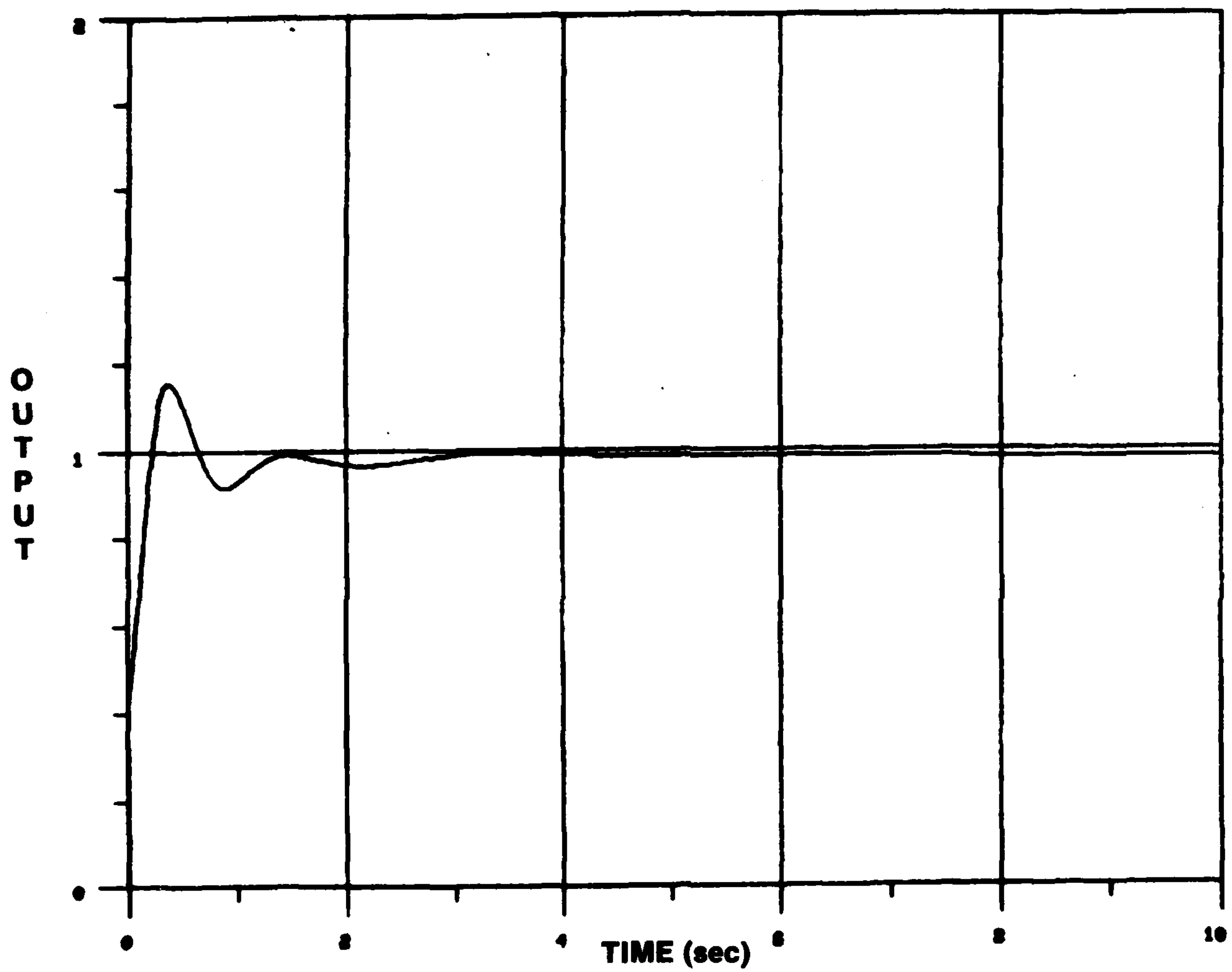

Figure 111. **Acceleration loop response, t_f = 9.85 sec, PGO = 1, W_1 = 1.0 - 0.2t, W_3 = 0.5 + 0.5t.**

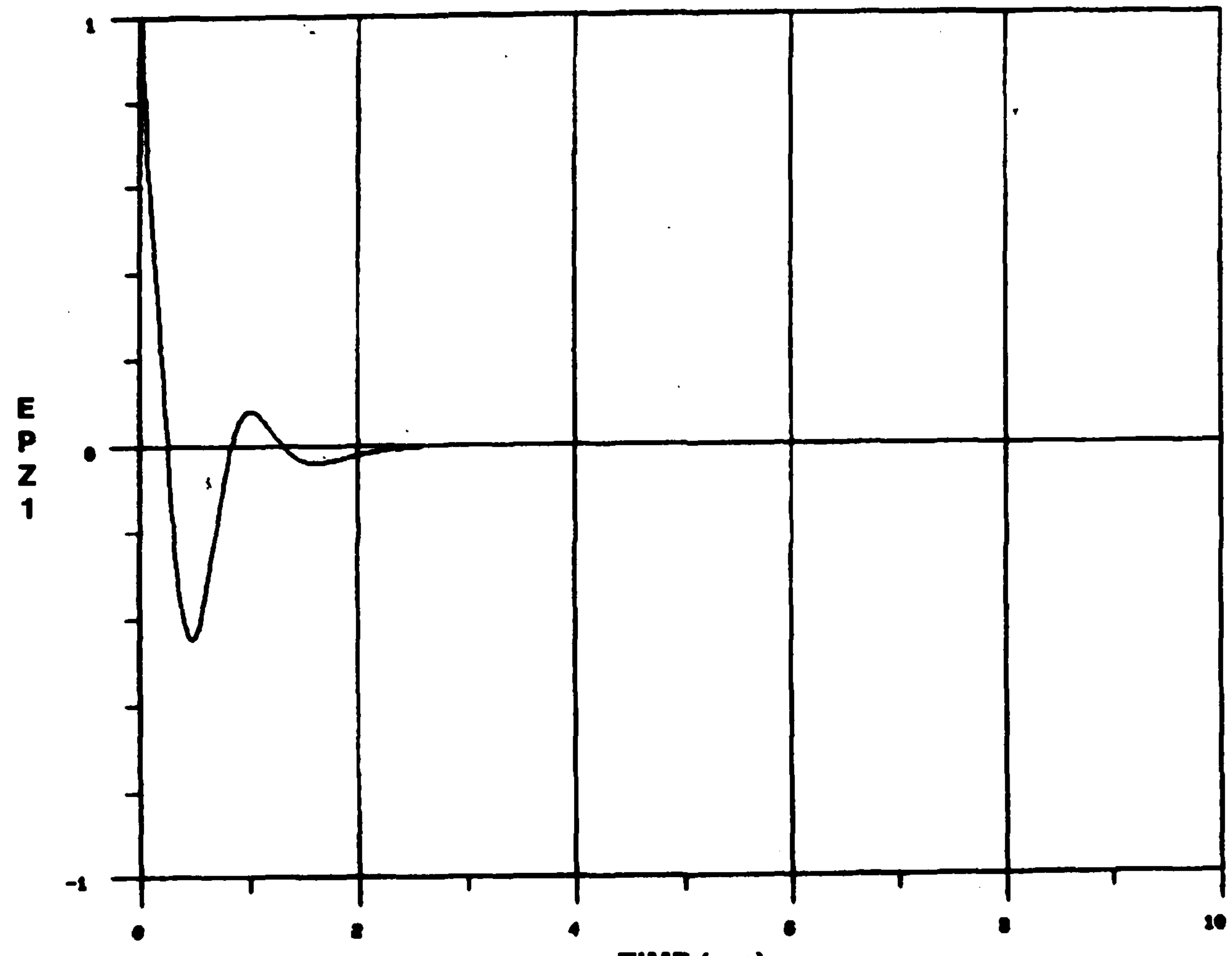

Figure 112. DAC disturbance estimation error for W_1, t_f = 9.85 sec, PGO = 1, W_1 = 1.0 - 0.2t, W_3 = 0.5 + 0.5t.

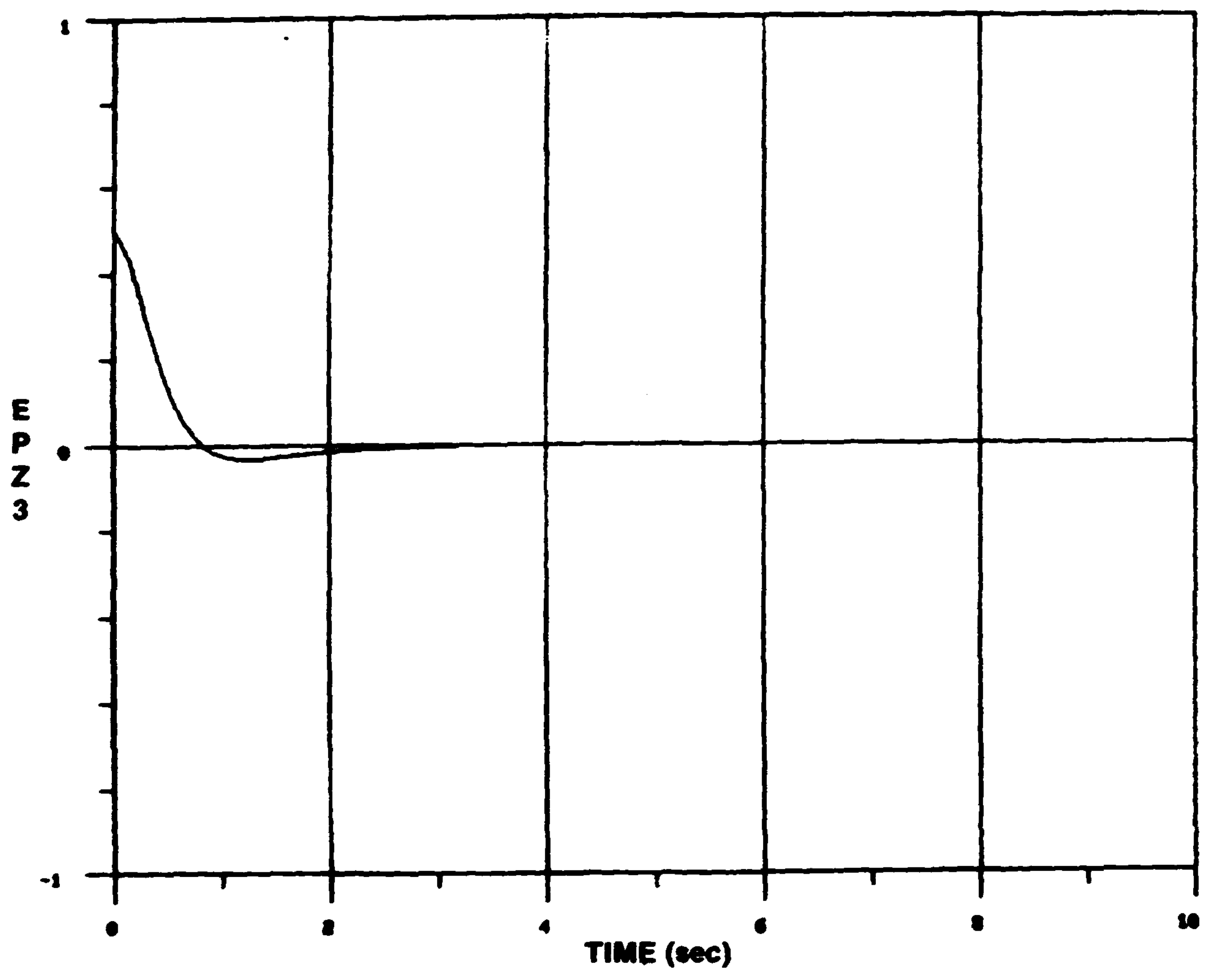

Figure 113. DAC disturbance estimation error for W_3, t_f = 9.85 sec, PGO = 1, W_1 = 1.0 - 0.2t, W_3 = 0.5 + 0.5t.

REFERENCES

1. Johnson, C. D., "Accommodation of Disturbances in Optimal Control Problems," *Int. Journal Control*, Vol. 15, No. 2, 1972, pp. 209-231.

2. Johnson, C. D., "On Observers for Systems with Unknown and Inaccessible Inputs," *Int. Journal Control*, Vol. 21, No. 5, 1975, pp. 825-831.

3. Johnson, C. D., "Accommodation of External Disturbances in Linear Regulator and Servomechanism Problems," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, Vol. AC-16, No. 6, December 1971.

4. Johnson, C. D., "Algebraic Solution of the Servomechanism Problem with External Disturbances," *Transactions of American Society of Mechanical Engineers, Journal of Dynamics Systems, Measurements and Control*, March 1974, pp. 25-35.

5. McCowan, Wayne L., *Investigation of Disturbance Accommodating Controller Design*, U.S. Army Missile Research and Development Command, Redstone Arsenal, Alabama, Report No. T-78-65, July 1978.

APPENDIX A

DIGITAL SIMULATION OF ACCELERATION LOOP WITH DISTURBANCE ON INPUT

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FORTRAN IV-PLUS V02-51   12:40:50   08-MAY-79
/TR:BLOCKS/WR

```
      COMMON X1,X2,X3,X4,XM1,XM2,XM3,XM4,ZM1,ZM2
      COMMON XD1,XD2,XD3,XD4,XDM1,XDM2,XDM3,XDM4,ZDM1,ZDM2
      COMMON KUTTA,DT,NX
      DIMENSION BUFFER(4)
 3333 CONTINUE
      NX=10
      NP=0
      DT=1./200.
      IJI=1
      XD1=0.
      XD2=0.
      XD3=0.
      XD4=0.
      XDM1=0.
      XDM2=0.
      XDM3=0.
      XDM4=0.
      ZDM1=0.
      ZDM2=0.
      X1=0.
      X2=0.
      X3=0.
      X4=0.
      XM1=0.
      XM2=0.
      XM3=0.
      XM4=0.
      ZM1=0.
      ZM2=0.
      X5=0.
      X6=0.
      XD5=0.
      XD6=0.
      XB1=0.
      XB2=0.
      DELTC=0.
C   33 AERO TF PARAMETERS AT DESIGN POINT
      ZETA=0.0695
      ZETADP=0.0695
      OMEGA=14.54
      OMGADP=14.54
      XKTD=-107.
      XKTDP=-107.
      GAM=0.536
      GAMDP=0.536
      OMEGMG=-540.
      OMGMDP=-540.
      XKETA=-317.8
      XKTADP=-317.8
      CR=-0.1306
      CRDP=-0.1306
      XP=1.
      XPDP=1.
```

```
      ZZ=10.
      ZZDP=10.
      A=0.

FORTRAN IV-PLUS V02-51        12:40:50    08-MAY-79
           /TR:BLOCKS/WR

      ADP=0.
      CN=1.0471E-04
      CNDP=1.0471E-04
      XL1=-5.
      XL2=-6.
      XL5=-12.
      XL6=-15.
      XL31=-10.
      XL32=2.
      XL41=-10.
      XL42=2.
      C0=0.
      C1=0.
      PGO=1.
      WRITE(6,*)' ZETA,OMEGA,OMGAD,ZETAD,XPDP'
      READ(5,*)ZETA,OMEGA,OMGAD,ZETAD,XPDP
      WRITE(6,*)' ZZDP,XKTDP,GAMDP,OMGNSD,CNDP'
      READ(5,*)ZZDP,XKTDP,GAMDP,OMGNSD,CNDP
      WRITE(6,*)' CRDP,XKTADP,XP,ZZ,XKTD'
      READ(5,*)CRDP,XKTADP,XP,ZZ,XKTD
      WRITE(6,*)' GAM,OMEGNS,CN,CR,XKETA'
      READ(5,*)GAM,OMEGNS,CN,CR,XKETA
      WRITE(6,*)' XL1,XL2,XL5,XL6,C0'
      READ(5,*)XL1,XL2,XL5,XL6,C0
      WRITE(6,*)' XL31,XL32,XL41,XL42,C1,PGO'
      READ(5,*)XL31,XL32,XL41,XL42,C1,PGO
      WRITE (6,5020)
 5020 FORMAT(2H0,4H XL1,12X,4H XL2,12X,4HXL31,12X,4HXL32,12X,4HXL41,12X,
     14HXL42,12X,4H XL5,12X,4H XL6/)
      WRITE (6,5027)XL1,XL2,XL31,XL32,XL41,XL42,XL5,XL6
 5027 FORMAT(2X,8(G12.6,2X))
      WRITE (6,4611)
 4611 FORMAT(2H0,4HXK11,12X,4HXK21,12X,4HXK31,12X,4HXK41,12X,4HXK12,12X,
     14HXK22/)
      T2OMAD=2.*ZETAD*OMGAD
      OMGADS=OMGAD*OMGAD
      T2OMA=2.*ZETA*OMEGA
      OMEGAS=OMEGA*OMEGA
      A0DP=T2OMAD
      A1DP=T2OMAD+XKTDP*CRDP
      A2DP=OMGADS+XK.DP*CRDP*ADP
      A0=T2OMA
      A1=T2OMA+XKTD*CR
```

```
0097        A2=OMEGAS+XKT3*CR1A
0098        B0DP=2.*ZZDP
0099        B1DP=ZZDP**2+OMGNSD
0100        B2DP=2.*ZZDP*OMGNSD
0101        B3DP=OMGNSD*ZZDP**2
0102        B4DP=2.*XPDP+A1DP
0103        B5DP=XPDP**2+2.*XPDP*A1DP+A2DP
0104        B6DP=A1DP*XPDP**2+2.*XPDP*A2DP
0105        B7DP=A2DP*XPDP**2
0106        B0=2.*ZZ
0107        B1=ZZ*ZZ+OMEGNS
0108        B2=2.*ZZ*OMEGNS
0109        B3=OMEGNS*ZZ*ZZ
```

```
FORTRAN IV-PLUS V02-51          12:40:50    09-MAY-79
                /TR:BLOCKS/WR

0110        B4=2.*XP+A1
0111        B5=XP*XP+2.*XP*A1+A2
0112        B6=A1*XP*XP+2.*XP*A2
0113        B7=A2*XP*XP
0114        XLS0=XL31*XL41+XL32*XL42
0115        Y=0.
0116        J=0
0117        XK22=-XL1*XL2*XLS0*XL5*XL6/(XKETASCN*OMEGNS*ZZ**2)
0118        XK12=(1./(XKETASCN*OMEGNS*ZZ**2))*((-2.*ZZ*OMEGNS*XKETASCN*XK22)
           1+XL1*XL2*XL5*XLS0+XL1*XL2*XLS0*XL6+XL1*XL2*XL31*XL5*XL6+
           2XL1*XL2*XL41*XL5*XL6+XL1*XLS0*XL5*XL6+XL2*XLS0*XL5*XL6)
0119        XK11=-XKETASCN*XK12+2.*XP+A1+XL1+XL2+XL31+XL41+XL5+XL6
0120        XK21=-XKETASCN*XK22-2.*ZZ*XKETASCN*XK12+XP*XP+2.*XP*A1+A2-
           1XL1*(XL2+XL31+XL41+XL5+XL6)-XL2*(XL31+XL41+XL5+XL6)-XL31*(XL5+XL6)
           2-XLS0-XL41*(XL5+XL6)-XL5*XL6
0121        XK31=-XKETASCN*(ZZ**2+OMEGNS)-XK12+2.*ZZ*XKETASCN*XK22+A1*XP*XP+
           12.*XP*A2+XL1*XL2*(XL31+XL41+XL5+XL6)+XL1*XL31*(XL5+XL6)+
           2XL1*XLS0+XL1*XL41*(XL5+XL6)+XL1*XL5*XL6+XL2*XL31*(XL5+XL6)+
           3XL2*XLS0+XL2*XL41*(XL5+XL6)+XL2*XL5*XL6+XLS0*(XL5+XL6)
           4+XL31*XL5*XL6+XL41*XL5*XL6
0122        XK41=-XKETASCN*2.*ZZ*OMEGNS*XK12
           1-XKETASCN*(ZZ*ZZ+OMEGNS)*XK22+A2*XP*XP-XL1*XL2*XL31*(XL5+XL6
           2)-XL1*XL2*XLS0-XL1*XL2*XL41*(XL5+XL6)-XL1*XL2*XL5*XL6-XL1*XLS0*
           3(XL5+XL6)-XL1*XL31*XL5*XL6-XL1*XL41*XL5*XL6-XL2*XLS0*
           4(XL5+XL6)-(XL2*XL31+XL2*XL41+XLS0)*XL5*XL6
0123        WRITE (6,4612)XK11,XK21,XK31,XK41,XK12,XK22
0124   4612 FORMAT(2X,6(G12.6,2X))
0125        WRITE (6,5000)
0126   5000 FORMAT(2H0,4HEPZ1,12X,4H  Y ,12X,5HDELTC,12X,4HTIME/)
0127        E0=XK11-B4
0128        E1=XK21-B5
0129        E2=XK31-B6
0130        E3=XK41-B7
```

```
      E6=XKETA*CN*(XK11+B0-B4)
      E7=XKETA*CN*(XK21+B1-B5)
      E8=XKETA*CN*(XK31+B2-B6)
      E9=XKETA*CN*(XK41+B3-B7)
      TIME=0.
 1010 CONTINUE
      IF(TIME.GE.10.) GO TO 1000
      J=J+1
      DO 100 KUTTA=1,4
      U1=C0+C1*TIME
      U=PGO-ZH1-Y
      U1=U+U1
      XD4=XKTADP*CNDP*B3DP*U1-B7DP*Y
      XD3=X4+XKTADP*CNDP*B2DP*U1-B6DP*Y
      XD2=X3+XKTADP*CNDP*B1DP*U1-B5DP*Y
      XD1=X2+XKTADP*CNDP*B0DP*U1-B4DP*Y
      Y=X1+XKTADP*CNDP*U1
      XDH1=E0*XH1+XH2+E6*ZH1-XK11*Y+XKETA*CN*(XK11+B0-B4)*U
      XDH2=E1*XH1+XH3+E7*ZH1-XK21*Y+XKETA*CN*(XK21+B1-B5)*U
      XDH3=E2*XH1+XH4+E8*ZH1-XK31*Y+XKETA*CN*(XK31+B2-B6)*U
      XDH4=E3*XH1+E9*ZH1-XK41*Y+XKETA*CN*(XK41+B3-B7)*U
      ZDH1=XK12*XH1+XK12*XKETA*CN*ZH1+ZH2-XK12*Y+XK12*XKETA*CN*U
      ZDH2=XK22*XH1+XK22*XKETA*CN*ZH1-XK22*Y+XK22*XKETA*CN*U
```

FORTRAN IV-PLUS V02-51 12:40:50 09-MAY-79
/TR:BLOCKS/WR

```
      GO TO (30,60,30,40),KUTTA
   30 CONTINUE
      NP=NP-1
      IF(KUTTA.GT.1)GO TO 31
      IF(NP.GT.0)GO TO 31
      NP=10
      BUFFER(1)=TIME
      BUFFER(2)=Y
      EPZ1=U1-ZH1
      BUFFER(3)=EPZ1
      BUFFER(4)=DELTC
      WRITE(2)BUFFER
   31 CONTINUE
      TIME=TIME+.5*DT
   40 CONTINUE
   60 CALL RUNGK
  100 CONTINUE
      IF(J.LT.32) GO TO 1010
      EPX1=X1-XH1
      EPZ1=U1-ZH1
      WRITE (6,5010)EPZ1,Y,DELTC,TIME
 5010 FORMAT(2X,4(G12.6,2X))
      IJI=IJI+1
```

```
0177          J=0
0178          GO TO 1010
0179     1000 CONTINUE
0180          GO TO 3333
0181          END
```

FORTRAN IV-PLUS V02-51   12:40:50   09-MAY-79
/TR:BLOCKS/WR

PROGRAM SECTIONS

NUMBER	NAME		SIZE	ATTRIBUTES
1	SCODE1	006132	1581	RW,I,CON,LCL
2	SPDATA	000302	97	RW,D,CON,LCL
3	SIDATA	000314	102	RW,D,CON,LCL
4	SVARS	000566	187	RW,D,CON,LCL
5	STEMPS	000034	14	RW,D,CON,LCL
6	.0000.	000130	44	RW,D,OVR,GBL

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
A	R*4	4-000170	ADP	R*4	4-000174	A0	R*4	4-000[illegible]	A0DP	R*4	4-000304	A1	R*4	4-000[illegible]
A1DP	R*4	4-000310	A2	R*4	4-000330	A2DP	R*4	4-000314	B0	R*4	4-000374	B0DP	R*4	4-000334
B1	R*4	4-000400	B1DP	R*4	4-000340	B2	R*4	4-000404	B2DP	R*4	4-000344	B3	R*4	4-000410
B3DP	R*4	4-000350	B4	R*4	4-000414	B4DP	R*4	4-000354	B5	R*4	4-000420	B5DP	R*4	4-000360
B6	R*4	4-000424	B6DP	R*4	4-000364	B7	R*4	4-000430	B7DP	R*4	4-000370	CM	R*4	[illegible]
CMDP	R*4	[illegible]	CR	R*4	4-000140	CRDP	R*4	4-000144	C0	R*4	[illegible]	C1	R*4	[illegible]
DELTC	R*4	[illegible]	DT	R*4	6-000122	EPX1	R*4	[illegible]	EPZ1	R*4	[illegible]	E0	R*4	4-000476
E1	R*4	[illegible]	E2	R*4	[illegible]	E3	R*4	4-000512	E6	R*4	4-000516	E7	R*4	[illegible]
E8	R*4	[illegible]	E9	R*4	4-000532	GAM	R*4	4-000110	GAMDP	R*4	4-000114	IJ1	I*2	[illegible]
J	I*2	4-000444	KUTTA	I*2	6-000120	MP	I*2	[illegible]	MK	I*2	6-000126	OMEGA	R*4	[illegible]
OMEGAS	R*4	[illegible]	OMEGMS	R*4	4-000120	OMGAD	R*4	[illegible]	OMGADD	R*4	[illegible]	OMGMGD	R*4	4-000124
PGO	R*4	[illegible]	TIME	R*4	4-000536	TZOMA	R*4	[illegible]	TZOMAD	R*4	[illegible]	U	R*4	[illegible]
U1	R*4	[illegible]	U1	R*4	4-000542	XDM1	R*4	6-000070	XDM2	R*4	6-000074	XDM3	R*4	6-000100
XDM4	R*4	6-000104	XD1	R*4	6-000050	XD2	R*4	6-000054	XD3	R*4	[illegible]	XD4	R*4	[illegible]
XD5	R*4	4-000034	XD6	R*4	4-000040	XM1	R*4	[illegible]	XM2	R*4	[illegible]	XM3	R*4	[illegible]
XM4	R*4	6-000034	XKETA	R*4	4-000130	XKTADP	R*4	4-000134	XKTD	R*4	4-000100	XKTDP	R*4	4-000104

```
FORTRAN IV-PLUS V02-51              10:04:30     11-MAY-79
                    /TR:BLOCKS/WR

              SUBROUTINE RUNGK
              COMMON X,DX,KUTTA,DT,NX
              DIMENSION X(10),DX(10),XA(10),DXA(10)
              GO TO (10,30,50,70),KUTTA
           10 DO 20 I=1,NX
              XA(I)=X(I)
              DXA(I)=DT*DX(I)
           20 X(I)=X(I)+.5*DXA(I)
              RETURN
           30 TDT=2.*DT
              HDT=.5*DT
              DO 40 I=1,NX
              DXA(I)=DXA(I)+TDT*DX(I)
           40 X(I)=XA(I)+HDT*DX(I)
              RETURN
           50 DO 60 I=1,NX
              VDT=DT*DX(I)
              DXA(I)=DXA(I)+2.*VDT
           60 X(I)=XA(I)+VDT
              RETURN
           70 DO 80 I=1,NX
           80 X(I)=XA(I)+(DXA(I)+DT*DX(I))/6.
              RETURN
               END
```

FORTRAN IV-PLUS V02-51 10:04:30 11-MAY-79
/TR:BLOCKS/WR

PROGRAM SECTIONS

NUMBER	NAME	SIZE		ATTRIBUTES
1	$CODE1	000510	164	RW,I,CON,LCL
2	$PDATA	000012	5	RW,D,CON,LCL
4	$VARS	000136	47	RW,D,CON,LCL
6	.$$$$.	000130	44	RW,D,OVR,GBL

ENTRY POINTS

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
RUNGK		1-000030												

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
DT	R*4	6-000122	HDT	R*4	4-000126	I	I*2	4-000120	KUTTA	I*2	6-000120	NX	I*2	6-000126
TDT	R*4	4-000122	UDT	R*4	4-000132									

ARRAYS

NAME	TYPE	ADDRESS	SIZE		DIMENSIONS
DX	R*4	6-000050	000050	20	(10)
DXA	R*4	4-000050	000050	20	(10)
X	R*4	6-000000	000050	20	(10)
XA	R*4	4-000000	000050	20	(10)

APPENDIX B

DIGITAL SIMULATION OF RATE LOOP WITH DISTURBANCE ON OUTPUT

```
FORTRAN IV-PLUS V02-51          13:30:01    09-MAY-79
                   /TR:BLOCKS/WR

      C     RATE LOOP WITH DAC
            COMMON X1,X2,XH1,XH2,ZH1,ZH2
            COMMON XD1,XD2,XDH1,XDH2,ZDH1,ZDH2
            COMMON KUTTA,DT,NK
            INTEGER*2 LABEL1,LABEL2,LABEL6,LABEL7
            DIMENSION LABEL1(6),LABEL2(4),LABEL6(3),LABEL7(5)
            DIMENSION BUFFER(3)
            DATA LABEL1/79,85,84,80,85,84/
            DATA LABEL2/84,73,77,69/
            DATA LABEL6/90,72,48/
            DATA LABEL7/88,83,48,69,82/
      3333  CONTINUE
            XD1=0.
            XD2=0.
            XDH1=0.
            XDH2=0.
            ZDH1=0.
            ZDH2=0.
            X1=0.
            X2=0.
            XH1=0.
            XH2=0.
            ZH1=0.
            ZH2=0.
            TIME=0.
            NK=0
            NP=0
            DT=1./256.
            XL1=-3.
            XL2=-5.
            XL31=-4.
            XL41=-4.
            XL32=2.
            XL42=-2.
            CRCH=-6.
            IJI=1
            ZETA=0.0856
            OMEGA=14.54
            XKTD=-107.
            GAM=0.536
            CR=-0.1306
            Y=0.
            J=0
            UC=1.
            C0=0.
            C1=0.
            WRITE(6,*)' ******************************'
            WRITE(6,*)'  HIT CNTL/Z TO TERMINATE JOB'
            WRITE(6,*)' ******************************'
            WRITE(6,*)' XL1,XL2,XL31,XL41,XL32,XL42'
            READ(5,*)XL1,XL2,XL31,XL41,XL32,XL42
            WRITE(6,*)' CRCH,ZETA,OMEGA,XKTD,GAM,CR'
            READ(5,*)CRCH,ZETA,OMEGA,XKTD,GAM,CR
```

```
      WRITE(6,*)' UC,C0,C1'
      READ(5,*)UC,C0,C1
      XLSQ=XL31*XL41+XL32*XL42
```

FORTRAN IV-PLUS V02-51   13:30:01   09-MAY-79
/TR:BLOCKS/WR

```
      TZOMA=2.*ZETA*OMEGA
      OMEGAS=OMEGA*OMEGA
      XK22=-(XL1*XL2*XLSQ)/(CR*OMEGAS)
      XK12=(1./(CR*OMEGAS))*(-TZOMA*CR*XK22+(XL1+XL2)*XLSQ+
     1(XL1*XL2)*(XL31+XL41))
      XK11=(1./CR)*(TZOMA-XK12*CR+XL1+XL2+XL31+XL41)
      XK21=(-1./CR)*(TZOMA*CR*(XK12-XK22)+XK11*XK12*CR*CR
     1-OMEGAS+XLSQ+XL1*(XL31+XL41)+XL2*(XL31+XL41)+XL1*XL2)
      B0=XK11-TZOMA-XKTD*CR
      B1=XK11-XKTD*CR
      B2=XK21-OMEGAS-XKTD*CR*GAM
      B3=XK21-XKTD*CR*GAM
      B4=XK12
      B5=XK12
      B6=XK22
      B7=XK22
      WRITE (6,2000)
 2000 FORMAT(2H0,4HXK11,12X,4HXK12,12X,4HXK21,12X,4HXK22/)
      WRITE (6,2001)XK11,XK12,XK21,XK22
 2001 FORMAT(2X,4(G12.6,2X))
      WRITE (6,2002)
 2002 FORMAT(2H0,2HB0,12X,2HB1,12X,2HB2,12X,2HB3,12X,2HB4,12X,2HB6/)
      WRITE (6,2004)B0,B1,B2,B3,B4,B6
 2004 FORMAT(2X,6(G12.6,2X))
      WRITE (6,2005)
 2005 FORMAT(4H0,3HXL1,12X,3HXL2,12X,4HXL31,12X,4HXL32,12X,4HXL41,12X,4H
     1XL42,12X,4HCRCH/)
      WRITE (6,2032)XL1,XL2,XL31,XL32,XL41,XL42,CRCH
 2032 FORMAT(4X,7(G12.6,2X))
      WRITE (6,2012)
 2012 FORMAT(2H0,4HTIME,12X,2HX1,12X,3HXH1,12X,2HX2,12X,3HXH2,12X,2HU2,1
     12X,3HZH1,12X,1HV/)
      WRITE (6,2005)
 2005 FORMAT(2H0,3HZH2,12X,1HU,12X,4HXDH1,12X,4HXDH2,12X,4HZDH1,12X,4HZD
     1H2/)
 1010 CONTINUE
      IF(TIME.GE.10.) GO TO 1000
      J=J+1
      DO 100 KUTTA=1,4
      WR=C0+C1*TIME
      UR=UC-V-CRCH*Z1
      DELTA=UR
      XD2=XKTD*GAM*DELTA-OMEGAS*X1
```

```
      XD1=X2+XKTD*DELTA-TZOMA*X1
      Y=CR*(X1+W2)
      XDH1=(XK11*CR-TZOMA)*XH1+XH2+XK11*CR*ZH1-XK11*Y+XKTD*UR
      XDH2=(XK21*CR-OMEGAS)*XH1+XK21*CR*ZH1-XK21*Y+XKTD*GAM*UR
      ZDH1=XK12*CR*(XH1+ZH1)+ZH2-XK12*Y
      ZDH2=XK22*CR*(XH1+ZH1)-XK22*Y
      GO TO (30,60,30,40),KUTTA
   30 CONTINUE
      NP=NP-1
      IF(KUTTA.GT.1) GO TO 31
      IF(NP.GT.0) GO TO 31
      NP=10
      EPSY=UC-Y
```

```
FORTRAN IV-PLUS V02-51          13:30:01    08-MAY-79
                /TR:BLOCKS/WR
```

```
      EPZ1=W2-ZH1
      BUFFER(1)=TIME
      BUFFER(2)=Y
      BUFFER(3)=ZH1
      WRITE(2)BUFFER
   31 CONTINUE
      TIME=TIME+.5*DT
   40 CONTINUE
   60 CALL RUNGK
  100 CONTINUE
      IF(J.LT.32) GO TO 1010
      WRITE (6,2015)TIME,X1,XH1,X2,XH2,W2,ZH1,Y
 2015 FORMAT(2X,8(G12.6,2X))
      WRITE (6,2017)ZH2,U,XDH1,XDH2,ZDH1,ZDH2
 2017 FORMAT(2X,6(G12.6,2X))
      IJI=IJI+1
      J=0
      GO TO 1010
 1000 CONTINUE
 4000 CONTINUE
      GO TO 3333
      STOP

[STOP       ]
ERROR  26
W NO PATH TO THIS STATEMENT

      END
```

```
FORTRAN IV-PLUS V02-51          13:30:01    09-MAY-79
               /TR:BLOCKS/WR

PROGRAM SECTIONS

NUMBER   NAME     SIZE                 ATTRIBUTES

  1     $CODE1   003336   879          RW,I,CON,LCL
  2     $PDATA   000234    78          RW,D,CON,LCL
  3     $IDATA   000464   154          RW,D,CON,LCL
  4     $VARS    000322   105          RW,D,CON,LCL
  5     $TEMPS   000010     4          RW,D,CON,LCL
  6     .$$$$.   000070    28          RW,D,OVR,GBL

VARIABLES

 NAME   TYPE  ADDRESS    NAME   TYPE  ADDRESS    NAME   TYPE  ADDRESS    NAME   TYPE  ADDRESS    NAME   TYPE  ADDRESS

 B0     R*4   4-000226   B1     R*4   4-000232   B2     R*4   4-000236   B3     R*4   4-000242   B4     R*4   4-000246
 B5     R*4   4-000252   B6     R*4   4-000256   B7     R*4   4-000262   CR     R*4   4-000144   CRCH   R*4   4-000116
 C0     R*4   4-000162   C1     R*4   4-000166   DELTA  R*4   4-000276   DT     R*4   6-000062   EPSY   R*4   4-000302
 EPZ1   R*4   4-000306   GAM    R*4   4-000140   IJI    I*2   4-000122   J      I*2   4-000154   KUTTA  I*2   6-000060
 NP     I*2   4-000064   MX     I*2   6-000066   OMEGA  R*4   4-000130   OMEGAS R*4   4-000202   TIME   R*4   4-000060
 TZORA  R*4   4-000176   U      R*4   4-000316   UC     R*4   4-000156   UR     R*4   4-000272   W2     R*4   4-000266
 W3     R*4   4-000312   XDM1   R*4   6-000040   XDM2   R*4   6-000044   XD1    R*4   6-000030   XD2    R*4   6-000034
 XM1    R*4   6-000010   XM2    R*4   6-000014   XKTD   R*4   4-000134   XK11   R*4   4-000216   XK12   R*4   4-000212
 XK21   R*4   4-000222   XK22   R*4   4-000206   XLSQ   R*4   4-000172   XL1    R*4   4-000066   XL2    R*4   4-000072
 XL31   R*4   4-000076   XL32   R*4   4-000106   XL41   R*4   4-000102   XL42   R*4   4-000112   X1     R*4   6-000000
 X2     R*4   6-000004   Y      R*4   4-000150   ZDM1   R*4   6-000050   ZDM2   R*4   6-000054   ZETA   R*4   4-000124
 ZM1    R*4   6-000020   ZM2    R*4   6-000024

ARRAYS

 NAME   TYPE  ADDRESS       SIZE        DIMENSIONS

 BUFFER R*4   4-000044   000014     6   (3)

 2004'  3-000130      2005'  3-000342      2012'  3-000252      2015'  3-000422      2025'  3-000144
 2030'  3-000236      3333   1-000020      4000     **

FUNCTIONS AND SUBROUTINES REFERENCED

 RUNGK

TOTAL SPACE ALLOCATED = 004500  1192
```

```
FORTRAN IV-PLUS V02-51              08:54:31    10-MAY-79
                /TR:BLOCKS/WR

          SUBROUTINE RUNGK
          COMMON X,DX,KUTTA,DT,NX
          DIMENSION X(6),DX(6),XA(6),DXA(6)
          GO TO (10,30,50,70),KUTTA
       10 DO 20 I=1,NX
          XA(I)=X(I)
          DXA(I)=DT*DX(I)
       20 X(I)=X(I)+.5*DXA(I)
          RETURN
       30 TDT=2.*DT
          HDT=.5*DT
          DO 40 I=1,NX
          DXA(I)=DXA(I)+TDT*DX(I)
       40 X(I)=XA(I)+HDT*DX(I)
          RETURN
       50 DO 60 I=1,NX
          VDT=DT*DX(I)
          DXA(I)=DXA(I)+2.*VDT
       60 X(I)=XA(I)+VDT
          RETURN
       70 DO 80 I=1,NX
       80 X(I)=XA(I)+(DXA(I)+DT*DX(I))/6.
          RETURN
          END
```

```
FORTRAN IV-PLUS V02-51              08:54:31    10-MAY-79
                /TR:BLOCKS/WR

PROGRAM SECTIONS

NUMBER   NAME        SIZE              ATTRIBUTES

   1     $CODE1    000510   164       RW,I,CON,LCL
   2     $PDATA    000012     5       RW,D,CON,LCL
   4     $VARS     000076    31       RW,D,CON,LCL
   6     .$$$$.    000070    28       RW,D,OVR,GBL
```

APPENDIX C

DIGITAL SIMULATION OF ACCELERATION LOOP WITH DISTURBANCE ON OUTPUT

```
FORTRAN IV-PLUS V02-51          10:03:25    11-MAY-79
                  /TR:BLOCKS/WR

      C     ACCELERATION LOOP, DAC,WITH U3 ON OUTPUT
            COMMON X1,X2,X3,X4,XH1,XH2,XH3,XH4,ZH1,ZH2
            COMMON XD1,XD2,XD3,XD4,XDH1,XDH2,XDH3,XDH4,ZDH1,ZDH2
            COMMON KUTTA,DT,NX
            DIMENSION BUFFER(4)
            NX=10
            DT=1./256.
            NP=0
       3333 CONTINUE
            X1=0.
            X2=0.
            X3=0.
            X4=0.
            XH1=0.
            XH2=0.
            XH3=0.
            XH4=0.
            ZH1=0.
            ZH2=0.
            XD1=0.
            XD2=0.
            XD3=0.
            XD4=0.
            XDH1=0.
            XDH2=0.
            XDH3=0.
            XDH4=0.
            ZDH1=0.
            ZDH2=0.
            XP=1.
            ZZ=10.
            ZETA=0.0256
            OMEGA=14.54
            XKTD=-107.
            GAM=0.536
            OMEGNS=-540.
            XKETA=-317.8
            CR=-0.1300
            C0=0.
            C1=0.
            A=0.
            CM=1.0471E-04
            TIME=0.
            XL1=-5.
            XL2=-6.
            XL31=-10.
            XL41=-10.
            XL32=2.
            XL42=-2.
            XL5=-12.
            XL6=-15.
            CRCH=0.0
            UC=1.0
```

```
      Y=0.
      WRITE(6,*)'******************************'
      WRITE(6,*)'  HIT CNTL/Z TO TERMINATE JOB'
```

```
FORTRAN IV-PLUS V02-51          10:03:25    11-MAY-79
                /TR:BLOCKS/WR

      WRITE(6,*)'******************************'
      WRITE(6,*)' XL1,XL2,XL31,XL41,XL32,XL42,XL5,XL6'
      READ(5,*)XL1,XL2,XL31,XL41,XL32,XL42,XL5,XL6
      WRITE(6,*)' ZETA,OMEGA,XKTD,GAM,OMEGNS,CR,XKETA'
      READ(5,*)ZETA,OMEGA,XKTD,GAM,OMEGNS,CR,XKETA
      WRITE(6,*)' UC,C0,C1,CRCH'
      READ(5,*)UC,C0,C1,CRCH
      WRITE (6,2064)
 2064 FORMAT(2H0,5HTZOMA,12X,2HCR,12X,4HXKTD,12X,6HOMEGAS,12X,5HXKETA,12
     1X,3HGAM,12X,6HOMEGNS,12X,2HCN/)
      WRITE (6,2067)TZOMA,CR,XKTD,OMEGAS,XKETA,GAM,OMEGNS,CN
 2067 FORMAT(2X,8(G12.6,2X))
      XLSQ=XL31*XL41+XL32*XL42

      TZOMA=2.*ZETA*OMEGA
      OMEGAS=OMEGA*OMEGA
      A0=TZOMA
      A1=TZOMA+XKTD*CR
      A2=OMEGAS+XKTD*CR*A
      B0=2.*ZZ
      B1=ZZ*ZZ+OMEGNS
      B2=2.*ZZ*OMEGNS
      B3=OMEGNS*ZZ*ZZ
      B4=2.*XP+A1
      B5=XP*XP+2.*XP*A1+A2
      B6=A1*XP*XP+2.*XP*A2
      B7=A2*XP*XP
      XKCN=XKETA*CN
      XK22=-(XL1*XL2*XLSQ*XL5*XL6)/B7
      XK12=-(1./B7)*(B6*XK22-(XL1*XL2*XLSQ*(XL5+XL6)+XL1*XL2*XL5*XL6*
     1(XL31+XL41)+XLSQ*XL5*XL6*(XL1+XL2)))
      XK41=B7-B5*XK22-B6*XK12-(XL1*XL2*XLSQ+XL1*XL2*XL31*(XL5+XL6)+
     1XL1*XL2*XL41*(XL5+XL6)+XL1*XL2*XL5*XL6+XL1*XLSQ*(XL5+XL6)+
     2XL1*XL5*XL6*(XL31+XL41)+XL2*XLSQ*(XL5+XL6)+XL2*XL5*XL6*(
     3XL31+XL41)+XLSQ*XL5*XL6)
      XK31=B6-B4*XK22-B5*XK12+(XL1*XL2*(XL31+XL41+XL5+XL6)+XL1*
     1(XLSQ+XL31*XL5+XL31*XL6)+XL1*(XL41*XL5+XL41*XL6+XL5*XL6)+
     2XL2*(XLSQ+XL31*XL5+XL31*XL6)+XL2*XL41*(XL5+XL6)+XL2*XL5*XL6+
     3XLSQ*(XL5+XL6)+(XL31+XL41)*XL5*XL6)
      XK21=B5-B4*XK12-XK22-(XL1*(XL2+XL31+XL41+XL5+XL6)+XL2*(XL31+XL41+
     1XL5+XL6)+XLSQ+ .L31*(XL5+XL6)+XL41*(XL5+XL6)+XL5*XL6)
      XK11=B4-XK12+XL1+XL2+XL31+XL41+XL5+XL6
      WRITE (6,2002)
```

```
      2002 FORMAT(2H0,2HD0,12X,2HD1,12X,2HD2,12X,2HD3,12X,2HD4,12X,2HD5,12X,2
          1HD6,12X,2HD7/)
           WRITE (6,2004)D0,D1,D2,D3,D4,D5,D6,D7
           WRITE (6,2000)
      2000 FORMAT(2H0,4HXK11,12X,4HXK21,12X,4HXK31,12X,4HXK41,12X,4HXK12,12X,
          14HXK22,12X,4HCRCH/)
           WRITE (6,2001)XK11,XK21,XK31,XK41,XK12,XK22,CRCH
      2004 FORMAT(2X,8(G12.6,2X))
      2001 FORMAT(2X,7(G12.6,2X))
           WRITE (6,2025)
      2025 FORMAT(2H0,3HXL1,12X,3HXL2,12X,4HXL31,12X,4HXL32,12X,4HXL41,12X,4H
          1XL42,12X,3HXL5,12X,3HXL6/)
           WRITE (6,2032)XL1,XL2,XL31,XL32,XL41,XL42,XL5,XL6
      2032 FORMAT(4X,8(G12.6,2X))
```

```
FORTRAN IV-PLUS V02-51          10:03:25     11-MAY-79
                  /TR:BLOCKS/WR

           WRITE (6,2012)
      2012 FORMAT(2H0,4HTIME,12X,2HX1,12X,3HXH1,12X,2HU3,12X,3HZH1,12X,1HU,12
          1X,1HV/)
           WRITE (6,2050)
      2050 FORMAT(2H0,3HXD4,12X,3HXD3,12X,3HXD2,12X,3HXD1,12X,4HXDH1,12X,4HZD
          1H1,12X,4HZDH2/)
      1010 CONTINUE
           IF(TIME.GE.10.) GO TO 1000
           J=J+1
           DO 100 KUTTA=1,4

           U3=C0+C1*TIME
           U=UC-Y+CRCH*ZH1
           XD4=-B7*X1+XKCN*(B3-B7)*U
           XD3=-B6*X1+X4+XKCN*(B2-B6)*U
           XD2=-B5*X1+X3+XKCN*(B1-B5)*U
           XD1=-B4*X1+X2+XKCN*(B0-B4)*U
           Y=U3+X1+XKCN*U
           XDH1=(XK11-B4)*XH1+XH2+XK11*ZH1-XK11*Y+XKCN*(B0-B4+XK11)*U
           XDH2=(XK21-B5)*XH1+XH3+XK21*ZH1-XK21*Y+XKCN*(B1-B5+XK21)*U
           XDH3=(XK31-B6)*XH1+XH4+XK31*ZH1-XK31*Y+XKCN*(B2-B6+XK31)*U
           XDH4=(XK41-B7)*XH1+XK41*ZH1-XK41*Y+XKCN*(B3-B7+XK41)*U
           ZDH1=XK12*XH1+XK12*ZH1+ZH2-XK12*Y+XKCN*XK12*U
           ZDH2=XK22*XH1+XK22*ZH1-XK22*Y+XKCN*XK22*U
           GO TO (30,60,30,40),KUTTA
        30 CONTINUE
           NP=NP-1
           IF(KUTTA.GT.1) GO TO 31
           IF(NP.GT.0) GO TO 31
           NP=10
           BUFFER(1)=TIME
           BUFFER(2)=Y
```

```
          EPZ1=U3-ZH1
          BUFFER(3)=EPZ1
          BUFFER(4)=U
          WRITE(2)BUFFER
       31 CONTINUE
          TIME=TIME+.5*DT
       40 CONTINUE
       60 CALL RUNGK
      100 CONTINUE
          IF(J.LT.32) GO TO 1010
          WRITE (6,2015)TIME,X1,XH1,U3,ZH1,U,Y
     2015 FORMAT(2X,7(G12.6,2X))
          IJI=IJI+1
          J=0
          GO TO 1010
     1000 CONTINUE
     4000 CONTINUE
          GO TO 3333
          STOP

CSTOP      ]
ERROR  26
U NO PATH TO THIS STATEMENT

          END
```


FORTRAN IV-PLUS V02-51   10:03:25   11-MAY-79
/TR:BLOCKS/WR

PROGRAM SECTIONS

NUMBER	NAME	SIZE		ATTRIBUTES
1	$CODE1	005106	1315	RW,I,CON,LCL
2	$PDATA	000274	94	RW,D,CON,LCL
3	$IDATA	000542	[illegible]	RW,D,CON,LCL
4	$VARS	000330	108	RW,D,CON,LCL
5	$TEMPS	000014	6	RW,D,CON,LCL
6	.$$$$.	000130	44	RW,D,OVR,GBL

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
A	R*4	4-000076	A0	R*4	4-000202	A1	R*4	4-000206	A2	R*4	4-000212	B0	R*4	4-000216
B1	R*4	4-000222	B2	R*4	4-000226	B3	R*4	4-000232	B4	R*4	4-000236	B5	R*4	4-000242
B6	R*4	4-000246	B7	R*4	4-000252	CH	R*4	4-000102	CR	R*4	4-000062	CRCH	R*4	4-000152
C0	R*4	4-000066	C1	R*4	4-000072	DT	R*4	6-000122	EPZ1	R*4	4-000304	GAM	R*4	4-000046
IJI	I*2	4-000330	J	I*2	4-000312	KUTTA	I*2	6-000126	NP	I*2	4-000000	NK	I*2	6-000126

9a

OMEGA	R*4	4-000036	OMEGAS	R*4	4-000172	OMEGNS	R*4	4-000052	TIME	R*4	4-000106	TZOMA	R*4	4-000166
U	R*4	4-000300	UC	R*4	4-000156	U3	R*4	4-000314	XDH1	R*4	6-000070	XDH2	R*4	6-000074
XDH3	R*4	6-000100	XDH4	R*4	6-000104	XD1	R*4	6-000050	XD2	R*4	6-000054	XD3	R*4	6-000060
XD4	R*4	6-000064	XH1	R*4	6-000020	XH2	R*4	6-000024	XH3	R*4	6-000030	XH4	R*4	6-000034
XKCH	R*4	4-000006	XKETA	R*4	4-000056	XKTD	R*4	4-000042	XK11	R*4	4-000306	XK12	R*4	4-000066
XK21	R*4	4-000302	XK22	R*4	4-000062	XK31	R*4	4-000276	XK41	R*4	4-000072	XL60	R*4	4-000176
XL1	R*4	4-000112	XL2	R*4	4-000116	XL31	R*4	4-000122	XL32	R*4	4-000132	XL41	R*4	4-000126
XL42	R*4	4-000136	XL5	R*4	4-000142	XL6	R*4	4-000146	XP	R*4	4-000002	X1	R*4	6-000000
X2	R*4	6-000004	X3	R*4	6-000010	X4	R*4	6-000014	Y	R*4	4-000162	ZDH1	R*4	6-000110
ZDH2	R*4	6-000114	ZETA	R*4	4-000032	ZH1	R*4	6-000040	ZH2	R*4	6-000044	ZZ	R*4	4-000026

ARRAYS

NAME	TYPE	ADDRESS	SIZE		DIMENSIONS
BUFFER	R*4	4-000000	000020	8	(4)

LABELS

LABEL	ADDRESS	LABEL	ADDRESS	LABEL	ADDRESS	LABEL	ADDRESS	LABEL	ADDRESS
30	1-004504	31	1-004640	40	1-004670	60	1-004676	100	**
1000	1-005074	1010	1-003376	2000'	3-000000	2001'	3-000316	2002'	3-000122
2004'	3-000302	2012'	3-000446	2015'	3-000514	2025'	3-000332	2032'	3-000432
2050'	3-000524	2054'	3-000000	2057'	3-000106	3333	1-000056	4000	**

FUNCTIONS AND SUBROUTINES REFERENCED

RUNGK

TOTAL SPACE ALLOCATED = 006742 1777

FORTRAN IV-PLUS V02-51 10:04:30 11-MAY-79
/TR:BLOCKS/WR

```
      SUBROUTINE RUNGK
      COMMON X,DX,KUTTA,DT,NX
      DIMENSION X(10),DX(10),XA(10),DXA(10)
      GO TO (10,30,50,70),KUTTA
   10 DO 20 I=1,NX
      XA(I)=X(I)
      DXA(I)=DT*DX(I)
   20 X(I)=X(I)+.5*DXA(I)
      RETURN
   30 TDT=2.*DT
      HDT=.5*DT
```

```
      DO 40 I=1,NX
      DXA(I)=DXA(I)+TDT*DX(I)
   40 X(I)=XA(I)+HDT*DX(I)
      RETURN
   50 DO 60 I=1,NX
      UDT=DT*DX(I)
      DXA(I)=DXA(I)+2.*UDT
   60 X(I)=XA(I)+UDT
      RETURN
   70 DO 80 I=1,NX
   80 X(I)=XA(I)+(DXA(I)+DT*DX(I))/6.
      RETURN
       END
```

```
FORTRAN IV-PLUS V02-51          10:04:30    11-MAY-79
                /TR:BLOCKS/WR
```

PROGRAM SECTIONS

NUMBER	NAME	SIZE		ATTRIBUTES
1	$CODE1	000510	164	RW,I,CON,LCL
2	$PDATA	000012	5	RW,D,CON,LCL
4	$VARS	000136	47	RW,D,CON,LCL
6	.$$$$.	000130	44	RW,D,OVR,GBL

ENTRY POINTS

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
RUNGK		1-000000												

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
DT	R*4	6-000122	HDT	R*4	4-000126	I	I*2	4-000120	KUTTA	I*2	6-000120	NX	I*2	6-000126
TDT	R*4	4-000122	UDT	R*4	4-000132									

ARRAYS

NAME	TYPE	ADDRESS	SIZE		DIMENSIONS
DX	R*4	6-000050	000050	20	(10)
DXA	R*4	4-000050	000050	20	(10)
X	R*4	6-000000	000050	20	(10)
XA	R*4	4-000000	000050	20	(10)

APPENDIX D

DIGITAL SIMULATION OF COMPOSITE ACCELERATION LOOP WITH DISTURBANCE ON INPUT AND OUTPUT

```
FORTRAN IV-PLUS V02-51           09:10:30    23-MAY-79
              /TR:BLOCKS/WR

        C     ACCEL LOOP WITH BOTH U1 AND U3
0001          COMMON X1,X2,X3,X4,X3H1,X3H2,X3H3,X3H4,Z3H1,Z3H2,XH1,XH2,XH3,XH4,
             1ZH1,ZH2
0002          COMMON XD1,XD2,XD3,XD4,XD3H1,XD3H2,XD3H3,XD3H4,ZD3H1,ZD3H2,
             1XDH1,XDH2,XDH3,XDH4,ZDH1,ZDH2
0003          COMMON KUTTA,DT,NX
0004          DIMENSION BUFFER(4)
0005          NX=16
0006          DT=1./256.
0007          NP=0
0008     3333 CONTINUE
0009          X1=0.
0010          X2=0.
0011          X3=0.
0012          X4=0.
0013          X3H1=0.
0014          X3H2=0.
0015          X3H3=0.
0016          X3H4=0.
0017          XH1=0.
0018          XH2=0.
0019          XH3=0.
0020          XH4=0.
0021          Z3H1=0.
0022          Z3H2=0.
0023          ZH1=0.
0024          ZH2=0.
0025          XDH3=0.
0026          XDH4=0.
0027          XD1=0.
0028          ZDH1=0.
0029          XD2=0.
0030          ZDH2=0.
0031          XD3=0.
0032          XD4=0.
0033          XD3H1=0.
0034          XD3H2=0.
0035          XD3H3=0.
0036          XD3H4=0.
0037          ZD3H1=0.
0038          ZD3H2=0.
0039          XDH1=0.
0040          XDH2=0.
0041          XP=1.
0042          ZZ=10.
0043          ZETA=0.0256
0044          OMEGA=14.54
0045          XKTD=-107.
0046          GAM=0.536
0047          OMEGMS=-540.
0048          XKETA=-317.8
0049          CR=-0.1360
0050          A=0.
```

```
ADP=0.
ZETAD=0.0256
OMGAD=14.54
```

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/TR:BLOCKS/WR

```
XKTDP=-107.
CRDP=-0.1369
XPDP=1.
ZZDP=10.
GAMDP=0.536
OMGNSD=-540.
XKTADP=-317.8
CN=1.0471E-04
CNDP=1.0471E-04
C0=0.
C1=0.
C3=0.
C4=0.
A=0.
TIME=0.
X3L1=-2.
X3L2=-3.
X3L31=-5.
X3L41=-5.
X3L32=4.
X3L42=-4.
X3L5=-6.
X3L6=-6.
XL1=-5.
XL2=-6.
XL31=-10.
XL41=-10.
XL32=2.
XL42=-2.
XL5=-12.
XL6=-15.
CRCH=0.0
UC=1.0
Y=0.
WRITE(6,*)' ******************************'
WRITE(6,*)'  HIT CNTL/Z TO TERMINATE JOB'
WRITE(6,*)' ******************************'
WRITE(6,*)' XL1,XL2,XL31,XL41,XL32,XL42,XL5,XL6'
READ(5,*)XL1,XL2,XL31,XL41,XL32,XL42,XL5,XL6
WRITE(6,*)' X3L1,X3L2,X3L31,X3L41,X3L32,X3L42,X3L5,X3L6'
READ(5,*)X3L1,X3L2,X3L31,X3L41,X3L32,X3L42,X3L5,X3L6
WRITE(6,*)' ZETA,OMEGA,XKTD,GAM,OMEGNS,CR,XKETA'
READ(5,*)ZETA,OMEGA,XKTD,GAM,OMEGNS,CR,XKETA
```

```
0097      WRITE(6,*)' ZETAD,OMGAD,XKTDP,GAMDP,OMGNSD,CRDP,XKTADP'
0098      READ(5,*)ZETAD,OMGAD,XKTDP,GAMDP,OMGNSD,CRDP,XKTADP
0099      WRITE(6,*)' UC,C0,C1,C3,C4,CRCH'
0100      READ(5,*)UC,C0,C1,C3,C4,CRCH
0101      XLS0=XL31*XL41+XL32*XL42
0102      X3LS0=X3L31*X3L41+X3L32*X3L42
0103      XKCH=XKETA*CH
0104      TZOMAD=2.*ZETAD*OMGAD
0105      OMGASD=OMGAD*OMGAD
0106      TZOMA=2.*ZETA*OMEGA
0107      OMEGAS=OMEGA*OMEGA
0108      A0DP=TZOMAD
0109      A1DP=TZOMAD+XKTDP*CRDP
```

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```
0110      A2DP=OMGASD+XKTDP*CRDP*ADP
0111      A0=TZOMA
0112      A1=TZOMA+XKTD*CR
0113      A2=OMEGAS+XKTD*CR*A
0114      B0DP=2.*ZZDP
0115      B1DP=ZZDP**2+OMGNSD
0116      B2DP=2.*ZZDP*OMGNSD
0117      B3DP=OMGNSD*ZZDP**2
0118      B4DP=2.*XPDP+A1DP
0119      B5DP=XPDP**2+2.*XPDP*A1DP+A2DP
0120      B6DP=A1DP*XPDP**2+2.*XPDP*A2DP
0121      B7DP=A2DP*XPDP**2
0122      B0=2.*ZZ
0123      B1=ZZ*ZZ+OMEGNS
0124      B2=2.*ZZ*OMEGNS
0125      B3=OMEGNS*ZZ*ZZ
0126      B4=2.*XP+A1
0127      B5=XP*XP+2.*XP*A1+A2
0128      B6=A1*XP*XP+2.*XP*A2
0129      B7=A2*XP*XP
0130      X3K22=-(X3L1*X3L2*X3LS0*X3L5*X3L6)/B7
0131      X3K12=-(1./B7)*(B6*X3K22-(X3L1*X3L2*X3LS0*(X3L5+X3L6)+X3L1*X3L2*
         1X3L5*X3L6*(X3L31+X3L41)+X3LS0*X3L5*X3L6*(X3L1+X3L2)))
0132      X3K41=B7-B5*X3K22-B6*X3K12-(X3L1*X3L2*X3LS0+X3L1*X3L2*X3L31*(
         1X3L5+X3L6)+X3L1*X3L2*X3L41*(X3L5+X3L6)+X3L1*X3L2*X3L5*X3L6+
         2X3L1*X3LS0*(X3L5+X3L6)+X3L1*X3L5*X3L6*(X3L31+X3L41)+X3L2*
         3X3LS0*(X3L5+X3L6)+X3L2*X3L5*X3L6*(X3L31+X3L41)+X3LS0*X3L5*X3L6)
0133      X3K31=B6-B4*X3K22-B5*X3K12+(X3L1*X3L2*(X3L31+X3L41 +X3L5+X3L6)+
         1X3L1*(X3LS0+X3L31*X3L5+X3L31*X3L6)+X3L1*(X3L41*X3L5+X3L41*X3L6+
         2X3L5*X3L6)+X3L2*(X3LS0+X3L31*X3L5+X3L31*X3L6)+X3L2*X3L41*(X3L5+
         3X3L6)+X3L2*X3L5*X3L6+X3LS0*(X3L5+X3L6)+(X3L31+X3L41)*X3L5*X3L6)
```

```
0134          X3K21=B5-B4*X3K12-X3K22-(X3L1*(X3L2+X3L31+X3L41+X3L5+X3L6)+X3L2*(
             1X3L31+X3L41+X3L5+X3L6)+X3LSQ+X3L31*(X3L5+X3L6)+X3L41*(X3L5+X3L6)+
             2X3L5*X3L6)
0135          X3K11=B4-X3K12+X3L1+X3L2+X3L31+X3L41+X3L5+X3L6
0136          XK22=-XL1*XL2*XLSQ*XL5*XL6/(XKCN*OMEGNS*ZZ**2)
0137          XK12=(1./(XKCN*B3))*((-B2*XKCN*XK22)
             1+XL1*XL2*XLSQ*(XL5+XL6)+XL1*XL2*XL5*XL6*(XL31+XL41)+XLSQ*XL5*XL6*
             2(XL1+XL2))
0138          XK11=-XKCN*XK12+B4+XL1+XL2+XL31+XL41+XL5+XL6
0139          XK21=-XKCN*XK22-2.*ZZ*XKCN*XK12+B5-XL1*(XL2+XL31+XL41+XL5+XL6)-
             1XL2*(XL31+XL41+XL5+XL6)-XL31*(XL5+XL6)-XLSQ-XL41*(XL5+XL6)-XL5*XL6
0140          XK31=-XKCN*B1-XK12+2.*ZZ*XKCN*XK22+B6+XL1*XL2*(XL31+XL41+XL5+XL6)+
             1XL1*XL31*(XL5+XL6)+XL1*XLSQ+XL1*XL41*(XL5+XL6)+XL1*XL5*XL6
             2XL2*XL31*(XL5+XL6)+XL2*XLSQ+XL2*XL41*(XL5+XL6)+XL2*XL5*XL6+XLSQ*
             3(XL5+XL6)+XL31*XL5*XL6+XL41*XL5*XL6
0141          XK41=-XKCN*B2*XK12-XKCN*B1*XK22+B7-XL1*XL2*XL31*(XL5+XL6)-XL1*XL2*
             1XLSQ-XL1*XL2*XL41*(XL5+XL6)-XL1*XL2*XL5*XL6-XL1*XLSQ*(XL5+XL6)-
             2XL1*XL31*XL5*XL6-XL1*XL41*XL5*XL6-XL2*XLSQ*(XL5+XL6)-(XL2*XL31+
             3XL2*XL41+XLSQ)*XL5*XL6
0142          WRITE (6,2057)ZETA,CR,XKTD,OMEGA,XKETA,GAM,OMEGNS,CN
0143     2057 FORMAT(2X,8(G12.6,2X))
0144          WRITE (6,2067)ZETAD,OMGAD,XKTDP,CRDP,GAMDP,OMGNSD,XKTADP,CNDP
0145     2067 FORMAT(2X,8(G12.6,2X))
0146          WRITE(6,2070)XK11,XK21,XK31,XK41,XK12,XK22
0147     2070 FORMAT(2X,6(G12.6,2X))
```



```
FORTRAN IV-PLUS V02-51          08:10:30     23-MAY-79
                   /TR:BLOCKS/WR

0148          WRITE (6,2071)X3K11,X3K21,X3K31,X3K41,X3K12,X3K22
0149     2071 FORMAT(2X,6(G12.6,2X))
0150          WRITE (6,2085)
0151     2085 FORMAT(2H0,3HXL1,12X,3HXL2,12X,4HXL31,12X,4HXL32,12X,4HXL41,12X,4H
             1XL42,12X,3HXL5,12X,3HXL6/)
0152          WRITE (6,2057)XL1,XL2,XL31,XL32,XL41,XL42,XL5,XL6
0153          WRITE (6,4085)
0154     4085 FORMAT(2H0,4HX3L1,12X,4HX3L2,12X,5HX3L31,12X,5HX3L32,12X,5HX3L41,1
             12X,5HX3L42,12X,4HX3L5,12X,4HX3L6/)
0155          WRITE (6,2057)X3L1,X3L2,X3L31,X3L32,X3L41,X3L42,X3L5,X3L6
0156          WRITE(6,3070)
0157     3070 FORMAT(2H0,4HCRCH/)
0158          WRITE (6,3074)CRCH
0159     3074 FORMAT(2X,G12.6)
0160          E0=XK11-B4
0161          E1=XK21-B5
0162          E2=XK31-B6
0163          E3=XK41-B7
0164          E6=XKCN*(XK11+B0-B4)
0165          E7=XKCN*(XK21+I .-B5)
0166          E8=XKCN*(XK31+B2-B6)
```

```
      Y0=XKCN*(X(4)+B3-B7)
      WRITE (6,2010)
 2010 FORMAT(2H0,4HTIME,12X,2HX1,12X,3HXH1,12X,2HU1,12X,3HZH1,12X,2HU3,1
     12X,4HZ3H1,12X,1HY/)
      WRITE (6,2050)
 2050 FORMAT(2H0,2HUC,12X,1HU,12X,2HU1/)
 1010 CONTINUE
      IF(TIME.GE.10.) GO TO 1000
      J=J+1
      DO 100 KUTTA=1,4
      W1=C0+C1*TIME
      W3=C3+C4*TIME
      U=UC-ZH1-Y+CRCH*Z3H1
      U1=U+W1
      XKCNDP=XKTADP*CNDP
      XD4=XKCNDP*B3DP*U1-B7DP*(XKCNDP*U1+X1)
      XD3=X4+XKCNDP*B2DP*U1-B6DP*(XKCNDP*U1+X1)
      XD2=X3+XKCNDP*B1DP*U1-B5DP*(XKCNDP*U1+X1)
      XD1=X2+XKCNDP*B0DP*U1-B4DP*(XKCNDP*U1+X1)
      Y=X1+XKTADP*CNDP*U1+W3
      YPRIME=X3H1+XKCNDP*U1
      XDH1=E0*XH1+XH2+E6*ZH1-XK11*YPRIME+XKCN*(XK11+B0-B4)*U
      XDH2=E1*XH1+XH3+E7*ZH1-XK21*YPRIME+XKCN*(XK21+B1-B5)*U
      XDH3=E2*XH1+XH4+E8*ZH1-XK31*YPRIME+XKCN*(XK31+B2-B6)*U
      XDH4=E3*XH1+E9*ZH1-XK41*YPRIME+XKCN*(XK41+B3-B7)*U
      ZDH1=XK12*XH1+XK12*XKCN*ZH1+ZH2-XK12*YPRIME+XK12*XKCN*U
      ZDH2=XK22*XH1+XK22*XKCN*ZH1-XK22*YPRIME+XK22*XKCN*U
      XD3H1=(X3K11-B4)*X3H1+X3H2+X3K11*Z3H1-X3K11*Y+XKCN*(B0-B4+X3K11)
     1*U1
      XD3H2=(X3K21-B5)*X3H1+X3H3+X3K21*Z3H1-X3K21*Y+XKCN*(B1-B5+X3K21)
     1*U1
      XD3H3=(X3K31-B6)*X3H1+X3H4+X3K31*Z3H1-X3K31*Y+XKCN*
     1(B2-B6+X3K31)*U1
      XD3H4=(X3K41-B7)*X3H1+X3K41*Z3H1-X3K41*Y+XKCN*
     1(B3-B7+X3K41)*U1
```

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```
      ZD3H1=X3K12*X3H1+X3K12*Z3H1+Z3H2-X3K12*Y+XKCN*X3K12*U1
      ZD3H2=X3K22*X3H1+X3K22*Z3H1-X3K22*Y+XKCN*X3K22*U1
      GO TO (30,60,30,40),KUTTA
   30 CONTINUE
      NP=NP-1
      IF(KUTTA.GT.1) GO TO 31
      IF(NP.GT.0) GO TO 31
      NP=10
      BUFFER(1)=TIME
      BUFFER(2)=Y
      EPZ1=U1-ZH1
```

```
        EPZ3=U3-Z3H1
        BUFFER(3)=EPZ1
        BUFFER(4)=EPZ3
        WRITE(2)BUFFER
   31   CONTINUE
        TIME=TIME+.5*DT
   40   CONTINUE
   60   CALL RUNGK
  100   CONTINUE
        IF(J.LT.30) GO TO 1010
        WRITE (6,2015)TIME,X1,XH1,U1,ZH1,U3,Z3H1,Y
 2015   FORMAT(2X,8(G12.6,2X))
        WRITE (6,5025)UC,U,U1
 5025   FORMAT(2X,3(G12.6,2X))
        J=0
        GO TO 1010
 1000   CONTINUE
        GO TO 3333
        END
```

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/TR:BLOCKS/WR

PROGRAM SECTIONS

NUMBER	NAME	SIZE		ATTRIBUTES
1	$CODE1	010744	2290	RW,I,CON,LCL
2	$PDATA	000432	141	RW,D,CON,LCL
3	$IDATA	000470	156	RW,D,CON,LCL
4	$VARS	000664	218	RW,D,CON,LCL
5	$TEMPS	000040	16	RW,D,CON,LCL
6	.$$$$.	000210	68	RW,D,OVR,GBL

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
A	R*4	4-000066	ADP	R*4	4-000072	A0	R*4	4-000362	A0DP	R*4	4-000346	A1	R*4	4-000366
A1DP	R*4	4-000352	A2	R*4	4-000372	A2DP	R*4	4-000356	B0	R*4	4-000436	B0DP	R*4	4-000376
B1	R*4	4-000442	B1DP	R*4	4-000402	B2	R*4	4-000446	B2DP	R*4	4-000406	B3	R*4	4-000452
B3DP	R*4	4-000412	B4	R*4	4-000456	B4DP	R*4	4-000416	B5	R*4	4-000462	B5DP	R*4	4-000422
B6	R*4	4-000466	B6DP	R*4	4-000426	B7	R*4	4-000472	B7DP	R*4	4-000432	CM	R*4	4-000142
CMDP	R*4	4-000146	CR	R*4	4-000062	CRCH	R*4	4-000076	CRDP	R*4	4-000112	C0	R*4	4-000152
C1	R*4	4-000156	C3	R*4	4-000162	C4	R*4	4-000166	DT	R*4	6-000002	EPZ1	R*4	4-000654
EPZ3	R*4	4-000660	E0	R*4	4-000562	E1	R*4	4-000566	E2	R*4	4-000572	E3	R*4	4-000576
E6	R*4	4-000602	E7	R*4	4-000606	E8	R*4	4-000612	E9	R*4	4-000616	GAM	R*4	4-000046
GAMDP	R*4	4-000126	J	I*2	4-000622	KUTTA	I*2	6-000200	NP	I*2	4-000020	NX	I*2	6-000206
OMEGA	R*4	4-000036	OMEGAS	R*4	4-000342	OMEGNS	R*4	4-000052	OMGAD	R*4	4-000102	OMGASD	R*4	4-000332

```
      SUBROUTINE RUNGK
      DIMENSION X(16),DX(16),XA(16),DXA(16)
      GO TO (10,30,50,70),KUTTA
10    DO 20 I=1,NX
      XA(I)=X(I)
      DXA(I)=DT*DX(I)
20    X(I)=X(I)+.5*DXA(I)
      RETURN
30    TDT=2.*DT
      HDT=.5*DT
      DO 40 I=1,NX
      DXA(I)=DXA(I)+TDT*DX(I)
40    X(I)=XA(I)+HDT*DX(I)
      RETURN
50    DO 60 I=1,NX
      VDT=DT*DX(I)
      DXA(I)=DXA(I)+2.*VDT
60    X(I)=XA(I)+VDT
      RETURN
70    DO 80 I=1,NX
80    X(I)=XA(I)+(DXA(I)+DT*DX(I))/6.
      RETURN
      END
```

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PROGRAM SECTIONS

NUMBER	NAME	SIZE		ATTRIBUTES
1	$CODE1	000510	164	RW,I,CON,LCL
2	$PDATA	000012	5	RW,D,CON,LCL
4	$VARS	000426	139	RW,D,CON,LCL

ENTRY POINTS

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
RUNGK		1-000000												

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
DT	R*4	4-000406	HDT	R*4	4-000416	I	I*2	4-000402	KUTTA	I*2	4-000400	NX	I*2	4-000404
TDT	R*4	4-000412	VDT	R*4	4-000422									

ARRAYS

NAME	TYPE	ADDRESS	SIZE		DIMENSIONS
DX	R*4	4-000100	000100	32	(16)

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